THESIS TITLE

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Department of Mathematics
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Mirpur University of Science and Technology (MUST), Mirpur
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(AJK) Pakistan

CERTIFICATION

I hereby undertake that this research is an original one and no part of this thesis falls under plagiarism. If found otherwise, at any stage, I will be responsible for the consequences.

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DEDICATION

I would like to dedicate this thesis to my beloved parents, who taught me to trust in Allah, believe in hard work and that so much could be done with little. This is also dedicated to my supervisor, who taught me that work hard through determination, self-focus and discipline, you can accomplish anything.

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List of Abbreviations

NLEEs	Nonlinear Evolution Equations
NLPDEs	Nonlinear Partial Differential Equations
NLODE	Nonlinear ordinary differential Equation
ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations
D	Dimensional
NLS	Nonlinear Schrödinger
KDV	Korteweg-De Vries
S-KDV	Schamel Korteweg-De Vries
GERF	Generalized Exponential Rational Function
AEM	Auxiliary Equation Mapping

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I would like to thank the supreme power of **Allah Almighty** who is obviously the one has always guided me to work on the right path of life. It is the blessings of **Allah Almighty** and His prophet **Hazrat Muhammad (PBUH)** that have enabled me to bring me this thesis to end.

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Student's Name

ABSTRACT

In the present world of competition there is a race of existence in which those are having will come to forward succeeded. The thesis serves as a link between theocratic and practical work, with this willing I joined this particular thesis. In this thesis, we have explored the analytical solutions to some selected nonlinear evolution equations by employing some interesting approaches.

Chapter 01

PRELIMINARIES

1.1 INTRODUCTION

Partial differential equations are of extensive interest because of their connection with phenomena in the physical world. These equations are also used to formulate models of the most basic theories underlying physics and engineering. For examples, they are foundational in the modern scientific understanding of sound, heat, diffusion, electrostatics, electrodynamics, fluid dynamics, elasticity, general relativity, and quantum mechanics. Their field is vast in size and diversity.

1.2 BACKGROUND AND MOTIVATIONS

Most of the work has been done with the NLEEs for the last five decades and is growing day by day due to their wide variety of applications to many physical processes of nature that occur in the different fields of applied and physical sciences. NLEEs are generally used to determine a lot of numerous significant physical phenomena showing up in plasma material science, power, electro a hereditary and so on.

During the recent years, the exact and analytical solutions encouraged investigators to huge size of research work to examine the nonlinear models.

1.3 ANALYTICAL METHODS

In the last few decades, the analytical solutions to NLEEs have attracted much attention to the many researchers. In the current era of applied science and technology, both mathematician and physicist are mostly engaged in this area to establish supplementary approaches. In this study, we will use different analytical techniques to solve nonlinear models.

Here, we will demonstrate only those techniques which applied successfully in this thesis which are the generalized exponential rational function method [1–3].

1.3.1 The Classical Khater Method

First consider the NLPDE of the form

$$Z\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, ...\right) = 0, \tag{1.3.1}$$

where Z is the polynomial of $u(x,t)$.
Then we consider the traveling wave transformation of the form

$$u(x,y,t) = U(\xi) \qquad \xi = x - \omega t, \tag{1.3.2}$$

by using this transformation Eq. (1.3.1) reduce to nonlinear ODE of the form

$$Q\left(U, U', U'', U''', ...\right) = 0, \tag{1.3.3}$$

where Q is the polynomial in $U(\xi)$ along with its derivatives w.r.t ξ.
The solution of Eq. (1.3.3) is of the form

$$U(\xi) = \sum_{i=0}^{n} \lambda_i a^{if(\xi)}, \tag{1.3.4}$$

where the arbitrary constants λ_i, $i = 0, 1, ..., N$ are to be determined later. By homogenous balancing principle the positive integer n can be determined from Eq. (1.3.3).
The Functions $f(\xi)$ satisfy the following equation

$$f'(\xi) = \frac{1}{\log(a)}\left(\alpha a^{-f(\xi)} + \beta + \gamma a^{f(\xi)}\right). \tag{1.3.5}$$

Using Eq. (1.3.4) and its desired derivatives along with Eq.(1.3.5) into Eq. (1.3.3) and by collecting the coefficients of $a^{if(\xi)}$ and equating them to zero we get an algebraic system. This system can be solved with the help of computer package programs and the values of parameters can be obtained. By substituting all values of parameters into Eq. (1.3.4), we get the solutions of Eq. (1.3.1) given as [1].

1.4 STRUCTURE OF THE THESIS

The purpose of this thesis is to discuss several analytical methods for obtaining the solution of NLEE in various disciplines. We use a very simple transformation for description of various significant methods. The chapter one included introduction of NLEEs, background and motivation, analytical methods such as, the generalized exponential rational function method, classical khater method and the extended modified AEM method have been explained in detail.

1.5 SUMMARY

In this chapter, the brief introduction of NLEEs and different analytical methods namely the generalized exponential rational function method, classical khater method and the extended modified AEM method that are going to be utilized in this study have been explained.

Chapter 02

THE (2+1)-D CHAFEE-INFANTE MODEL

2.1 INTRODUCTION

The (2+1)-d Chafee-Infante model read as [**?**]

$$\left(\alpha u^3 - \alpha u - u_{\mathrm{xx}x}\right) + u_{\mathrm{xt}} + \sigma u_{\mathrm{yy}} = 0, \tag{2.1.1}$$

where α is the coefficient of diffusion and σ is the degradation coefficient.

2.2 MATHEMATICAL ANALYSIS

Consider the wave transformation

$$u(x, y, t) = u(\xi), \qquad \xi = x + y - ct, \tag{2.2.1}$$

where c is the speed of the travelling wave.

2.2.1 Applications

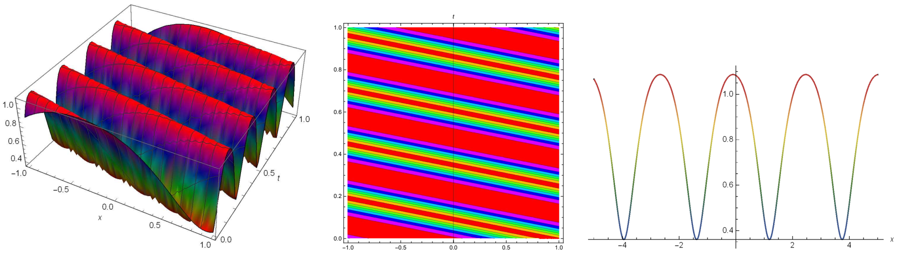
According to extended modified AEM method the solution is of the form

$$u = a_0 + a_1\psi(\xi) + \frac{b_1}{\psi(\xi)} + d_1\frac{\psi'(\xi)}{\psi(\xi)}, \tag{2.2.2}$$

where a_0, a_1, a_0, b_1 and d_1 are the constant to be determined.

2.3 GRAPHICAL REPRESENTATION

The 3D, contour and 2D graphs visualizes the nature of nonlinear waves constructed from Eq. (2.1.1).



(a) (b) (c)

Figure 2.1: $u_1(x, y, t):\ \alpha = 2,\ b_1 = 1,\ c = -10,\ y = 1,\ \delta_1 = -6,\ \delta_3 = 2.$

2.4 SUMMARY

In our work, the extended modified AEM method and the classical khater method are successfully employed to get some new exact solitary wave solutions of the (2+1)-d Chaffee-Infante equation.

Chapter 03

THE GENERALIZED NLS MODEL

3.1 INTRODUCTION

The main objective of this chapter is to attain some new exact solutions to the generalized coupled NLS-KdV equations by employing GERF method [**?**]. The generalized coupled NLS-KdV equations [**?**] read as

$$\begin{aligned} \lambda_2 P\left|P\right|^2 + \lambda_3 PQ + iP_t + \lambda_1 P_{\text{xx}} = 0, \\ \beta_3 \left|P\right|^2{}_x + Q_t + \beta_1 QQ_x + \beta_2 Q_{\text{xxx}} = 0, \end{aligned} \tag{3.1.1}$$

where $\beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2$ and λ_3 are real constants.

3.2 MATHEMATICAL ANALYSIS

Consider the transformation

$$\begin{aligned} P(x,t) = P(\phi)e^{i(\eta_2 t + x\omega_2)}, \\ Q(x,t) = Q(\phi),\ \phi = \eta_1 t + x\omega_1, \end{aligned} \tag{3.2.1}$$

where ω_i and η_i, i = 1, 2 are the speed of wave, wave number and frequency of the soliton respectively.

3.2.1 Applications

According to GERF method we assume the solution of Eq.(**??**) and Eq.(**??**) respectively as

$$\begin{aligned} P(\phi) = a_0 + \sum_{i=1}^{n} a_i \psi(\phi)^i + \sum_{i=1}^{n} \frac{b_i}{\psi(\phi)^i}, \\ Q(\phi) = c_0 + \sum_{i=1}^{m} c_i \psi(\phi)^i + \sum_{i=1}^{m} \frac{d_i}{\psi(\phi)^i}, \end{aligned} \tag{3.2.2}$$

CONCLUSIONS

The nonlinear evolution equations are extremely useful in present era of research and technology for illustrating a wide range of complex natural phenomena. Most of the real-world physical models are governed by the nonlinear evolution equations. Consequently, the problem for developing new techniques to solve such nonlinear models are very crucial because they are capable to elucidate the real features in mathematical physics.

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