7TH MEETING OF THE BOARD OF STUDIES

DEPARTMENT OF MATHEMATICS

WEDNESDAY, OCTOBER 18, 2023

PROPOSED CHANGES IN THE SCHEME OF STUDIES ADS, BS, BS SUCCEEDED BY ADS, MPHIL AND PHD PROGRAMS

MIRPUR UNIVERSITY OF SCIENCE AND TECHNOLOGY (MUST)

ALLAMA IQBAL ROAD, MIRPUR-10250

MIRPUR, AZAD JAMMU & KASHMIR

Members of the Board of Studies

The following members were present in the meeting:

Sr. #	Name	Designation	Signature
1	Prof. Dr. Rashida Hussain Coordinator, Department of Mathematics, MUST, Mirpur AJ&K	Convener	
2	Prof. Dr. Muhammad Mushtaq Dept. of Mathematics, UET Lahore	Member/Subject Expert	
3	Prof. Dr. Qasim Ali Chaudhary Dept. of Mathematics, UET Lahore	Member/Subject Expert	
4	Dr. Jamshed Ahmed Dept. of Mathematics, University of Gujarat	Member/Subject Expert	
5	Miss Rehanna Razaq Affiliated Colleges, Ghazi Ellahi Bhkash PGC, Mirpur AJK	Member	
6	Dr. Syed Zakir Hussain Bukhari, Associate Prof. Dept. of Mathematics, MUST, Mirpur AJK		
7	Dr. Kalim Ul Haq Tariq Associate Prof. Dept. of Mathematics, MUST, Mirpur AJK	Member	
8	The Controller of Examinations, MUST, Mirpur AJK	Member	
9	Representative of Alumni	Member	
10	Representative of Industry	Member	
11	The Registrar/Nominee, MUST, Mirpur AJK	Observer	
12	Director QEC/Nominee, MUST, Mirpur AJK	Observer	
13	Chairman, Semester Committee Dept. of Mathematics, MUST, Mirpur AJK	Secretary	

Introduction

The department of mathematics at MUST, Mirpur, AJ&K, was established in 2009. The first intake of the M.Sc program was inducted in the Spring 2010. The MPhil program was launched in the Spring 2011. In the Fall 2012, the department launched its BS program. The PhD program was started in the Spring 2015.

Mission Statement: The department of mathematics is committed to provide a supportive and condusive academic environment for learning through its various programs. The focus of these programs is to produce intellectual mathematicians equipped with critical thinking, problem solving skills, innovation, and high professional ethics who could bring positive changes in our society.

1(a) Scheme of Studies for BS Mathematics: General Breakup

Content		Description	Remarks
1.1	Awarding Institute/Body	Mirpur University of Science and Technology (MUST)	
1.2	Teaching Institute	Department of Mathematics, Mirpur University of Science and Technology (MUST), and affiliated colleges	
1.3	Final Award	Bachelor Studies in Mathematics or Bachelor of Science in Mathematics with an abbreviation of BS in Mathematics. Decide according to Uniform MUST policy	
1.5	Starting Time for Program	Fall/Spring semester of each academic year	
1.6	Duration of the Program	8-14 Semesters	
1.7	Entrance Requirements	Intermediate or equivalent degree with mathematics (Min 45% marks)	
		No D-grade/3rd Division in matric and intermediate	
		Entry Test conducted by the University with the following breakup: Mathematics: 30 %, English: 10%, two other subjects which the candidate have studied in intermediate with 30% weightage for each	
1.8	Merit Formula	Merit shall be determined on 20% of SSC, 50% of Intermediate and 30% of Entry Test marks.	
1.9	Total Credit Hours	Course Work: 125 Credit Hrs	
		Capstone/ Project (Compulsory): 3 Credit Hrs	
		Field Experience /Internship(Compulsory): 3 Credit Hrs	
		8Compulsory Courses Holy Quran with Translation S/U	

1.10 Program Educational Objectives:

The BS Mathematics program is aimed at imparting quality education of mathematics to the youth of Mirpur division and the surrounding districts, at an affordable cost. The program will give an opportunity to the talented youth to satiate their desire to learn and excel in mathematics. The program will meet the demand of skilled mathematicians in the local job market.

The objectives of the BS program also include: to teach students basic concepts of mathematics, to empower them with analytical and computational skills, to develop critical thinking, and to develop professional approach and work ethics. Moreover, the program is designed in such a way that the students can learn advanced mathematical concepts through the course work and the semester projects.

1.11 Program Learning Outcomes (PLOs):

The curriculum of the BS program is very diverse. During the first 4 semesters, beside some of the fundamental mathematics courses, the students also take many general and compulsory courses recommended by HEC new Education Policy like Quantitative Reasoning I, Quantitative Reasoning II, Civics and Community Engagement, Ideology and constitution of Pakistan, etc. These courses are so designed that students get useful knowledge and skills.

1.12 Scope of the Program:

The mathematical experts are in demand across all kind of industries, the world over. Our graduates will be able to seek career opportunities in:

a). Teaching at school, college, or higher level

b). R&D and strategic organizations like PAEC, NESCOM, SUPARCO, etc.

c). Banking sector, trading, and stock exchange businesses

d). Higher studies in national and international universities and institutes

e). Armed forces, civil services, oil and gas sector,

1.13 **Structure**

Sr. No.	Category	No. of Courses	Credit Hrs	Remarks
01	General Education Cluster	12	30	
02	Allied Courses	04	12	
03	Major Courses	**29**	83	
04	Field Experience	01	03	
05	Capstone Project	01	03	
06	Compulsory Courses	08	Non	S/U basis
Total		53	131	

1.14 Layout/Framework

Category	Course Title	Credit Hrs	Remarks
General Education Cluster	Introduction to Microbiology	2+1	**30**
	Arabic	02	
	Introduction to Sociology/HR Management	02	
	Functional English	03	
	Expository Writing	03	
	Quantitative Reasoning I	03	
	Quantitative Reasoning II	03	
	Islamic Studies	02	
	Ideology and constitution of Pakistan	02	
	Application of information and communication technologies	03	
	Entrepreneurship	02	
	Civics and Community Engagement	02	
Allied Courses	Mathematical Statistics I	03	12 Credits
	Mathematical Statistics II	03	
	Physics-I	03	
	Educational Psychology/Organizational Behavior/IIR/ Intro Law	03	
Major Courses	Calculus-I	03	**83**
	Calculus-II	03	
	Calculus-III	03	
	Linear Algebra-I	03	
	Linear Algebra-II	02	
	Differential Equations I	02	
	Mechanics I	03	
	Mechanics II	03	
	Differential Equations II	03	
	Abstract Algebra	03	
	Real Analysis-I	03	
	Complex Analysis	03	
	Metric Spaces	02	
	Topology	03	
	Scientific Programming	2+1	
	Tensor Analysis	02	
	Differential Geometry-I	03	

	Real Analysis-II	03		
	Partial Differential Equations	03		
	Analytical Mechanics	03		
	Numerical Methods I	03		
	Mathematical Physics	03		
	Functional Analysis	03		
	Integral Equations	03		
	Elective-I	03		
	Elective-II	03		
	Elective-III	03		
	Elective-IV	03		
	Elective-V	03		
Compulsory	Field Experience	03		
	Capston/ Project			
	Holy Quran with Translation, Tajveed and Tafseer	S/U	03	
	Capstone Project	03	03	

1.15 Semester-Wise Breakdown

Course Code	Course Title	Lec. Hrs.	Lab. Hrs.	Credit Hrs.
	1st Year			
Semester-I				
MATH-1101	Calculus-I	3	0	3
BIOT-1102	Introduction to Microbiology	2	1	3
ENG-1107	Functional English	3	0	3
ISL-1112	Islamic Studies	2	0	2
QTR- 1124	Quantitative Reasoning 1	3	0	3
ICT-1126	Application of information and communication technologies	2	1	3
HQT-1128	Holy Quran with Translation			S/U
Total Credit Hr				**17**
Semester-II				
MATH-1201	Calculus-II	3	0	3
MATH- 1202	Linear Algebra-I	3	0	3
ENG-1207	Expository Writing	3	0	3
PHY-1218	Physics-I	3	0	3
QTR-1224	Quantitative Reasoning II	3	0	3
ICP-1227	Ideology and Constitution of Pakistan	2	0	2
Total Cr Hrs				**17**
	2nd Year			
Semester-III				
MATH-2301	Calculus-III	3	0	3
MATH-2302	Linear Algebra-II	2	0	2
MATH-2303	Mathematical Statistics-I	3	0	3
MATH-2304	Mechanics-I	3	0	3
CCE-2325	Civic and Community Engagement	2	0	2
	One Optional			
PSY-2319	Introduction to Sociology	2	0	2
HRM-2309	HR Management	2	0	2
EC-2330	Introduction to Environmental Sciences	2	0	2

				15
Total Credit Hrs				
Semester-IV				
ARA-2401	Arabic	2	0	2
MATH-2402	Differential Equations I	2	0	2
MATH-2403	Mathematical Statistics II	3	0	3
MATH-2404	Mechanics-II	3	0	3
MATH-2405	Metric Spaces	2	0	2
ETR-2408	Entrepreneurship	2	0	2
	One Optional			
MATH-2406	Business Mathematics	3	0	3
IR-2410	Introduction to International Relations	3	0	3
LAW-2411	Introduction to Law	3	0	3
OB-2416	Organizational Behavior	3	0	3
PSY-2419	Educational Psychology	3	0	3
Total Credit				**17**
	3rd Year			
Semester-V				
MATH-3501	Real Analysis-I	3	0	3
MATH-3502	Differential Equations II	3	0	3
MATH-3503	Differential Geometry-I	3	0	3
MATH-3504	Analytical Mechanics	3	0	3
MATH-3505	Topology	3	0	3
MATH-3506	Abstract Algebra	3	0	3
Total Credit	**Hrs**			**18**
Semester-VI				
MATH-3601	Real Analysis-II	3	0	3
MATH-3602	Partial Differential Equations	3	0	3
MATH-3603	Numerical Methods-I	3	0	3

MATH-3604	Complex Analysis	3	0	3
MATH-3605	Tensor Analysis	2	0	2
MATH-3606	Scientific Programming	2	1	3

17

Total

4th Year

Semester-VII

MATH-4701	Mathematical Physics	3	0	3
MATH-4702	Functional Analysis	3	0	3
MATH-4703	Internship /Field Experience	3	0	3

Two Optional Courses

MATH-4704	Measure Theory	3	0	3
MATH-4705	Numerical Methods II	3	0	3
MATH-4706	Fluid Mechanics-I	3	0	3
MATH-4707	Discrete Structures	3	0	3
MATH-4708	Special Functions	3	0	3
MATH-4709	Quantum Mechanics-I	3	0	3
MATH-4710	Ring Theory	3	0	3
MATH-4711	Analytical Dynamics	3	0	3
MATH-4712	Approximation Theory	3	0	3
MATH-4713	Differential Geometry-II	3	0	3
MATH-4714	Electromagnetic Theory-I	3	0	3
MATH-4715	Introduction to Univalent Functions	3	0	3

Semester-VIII

MATH-4801	Integral Equations	3	0	3
MATH-4802	Capstone Project	3	0	3

Three Optional Courses

MATH-4803	Optimization Theory	3	0	3
MATH-4804	Fluid Mechanics-II	3	0	3
MATH-4805	Mathematical Modeling	3	0	3
MATH-4806	Fractional Calculus	3	0	3

MATH-4807	Algebraic Topology	3	0	3
MATH-4808	Mathematical System Theory	3	0	3
MATH-4809	Dynamical Systems	3	0	3
MATH-4810	Quantum Mechanics-II	3	0	3
MATH-4811	Introduction to Combinatorics	3	0	3
MATH-4812	Variational Inequalities	3	0	3
MATH-4813	Theory of Elasticity	3	0	3
MATH-4814	Electromagnetic Theory-II	3	0	3
MATH-4815	Special Theory of Relativity	3	0	3

1.16 Detail of Courses

Semester-I

Course Code: MATH-1101 **Title:** Calculus-I **Credit Hrs:** 03

Course Outline:

Review Topics: Equations and inequalities**:** Solving linear and quadratic equations, linear inequalities. Division of polynomials, synthetic division. Roots of a polynomial, rational roots; Viete Relations. Descartes rule of signs. Solutions of equations with absolute value sign. Solution of linear and non-linear inequalities with absolute value sign.

Functions and Graphs: Domain and range of a function. Examples: polynomial, rational, piecewise defined functions, absolute value, functions, and evaluation of such functions. Operations with functions: sum, product, quotient and composition. Graphs of functions: linear, quadratic, piecewise defined functions.

Lines and System of Equations: Equation of a straight line, slope and intercept of a line, parallel and perpendicular lines. Systems of linear equations, solution of system of linear equations. Nonlinear systems: at least one quadratic equation.

Limits and Continuity: Functions, limit of a function. Graphical approach. Properties of limits. Theorems of limits. Limits of polynomials, rational and transcendental functions. Limits at infinity, infinite limits, one-sided limits. Continuity.

Derivatives: Definition, techniques of differentiation. Derivatives of polynomials and rational, exponential, logarithmic and trigonometric functions. The chain rule. Implicit differentiation. Rates of change in natural and social sciences. Related rates. Linear approximations and differentials. Higher derivatives, Leibnitz's theorem.

Applications of derivatives: Roll's Theorem, Cauchy's MVT, Increasing and decreasing functions. Relative extrema and optimization. First derivative test for relative extrema. Convexity and point of inflection. The second derivative test for extrema. Curve sketching. Indeterminate forms and L'Hopitals rule. Inverse functions and their derivatives.

Integration: Antiderivatives and integrals. Riemann sums and the definite integral. Properties of definite integral. The fundamental theorem of calculus. The substitution rule.

Texts and Reference Books

1. H. Anton, *Calculus, A New Horizon,* 6th Edition, John Wiley and Sons, New York, 1999
2. J. Stewart, *Calculus,* 3rd Edition, Brooks/Cole, 1995.
3. G. B. Thomas and A.R. Finney, *Calculus,* 10th Edition, Addison Wesley, USA, 2002.
4. Dr. B. H. Edwards, *Calculus*: 10th Edition, Brooks/Cole, 2013.
5. S.M Yousaf, Muhammad Amin, *Calculus*: 7th Edition, Ilmi Kitab Khana.

Course Code: BIOT-1102 **Title:** Introduction to Microbiology **Credit Hrs:** 2+1

CONTENTS:
Overview and history of microbiology including microbial diversity (Archaea, bacteria, fungi, algae, protozoa), nutrition, growth, metabolism; cultivation; viruses; control of microorganisms: sterilization and disinfection, antimicrobial agents, antibiotics, antibiotic resistance and susceptibility, antifungal and antiviral agents; cell death; symbiosis, carbon, nitrogen, sulfur and phosphorus cycles; microbiology of soil, freshwater and seawater.

PRACTICALS:
Sterilization techniques; culturing of bacteria in liquid and on solid medium; Gram-staining of bacteria; colony and cell morphology; bacterial cell count and growth curves; biochemical tests.

RECOMMENDED BOOKS:

1. Alcamo, I. E. 2010. Fundamentals of Microbiology. Ninth Edition, Jones and Bartlett Publishers.
2. Madigan, M. T. and J. Martinko. 2010. Brock Biology of Microorganisms. 13th Edition; Pearson College Div.
3. Talaro, K. P. 2009. Foundations in Microbiology: Basic Principles. Seventh Edition; McGraw Hill Publisher.
4. Black, J. G. 2007. Microbiology: principles and explorations. Seventh Edition; John Wiley and Sons.
5. Baker *et al*., 2006. Instant Notes in Microbiology. Third Edition; Taylor and Francis.
6. Prescott *et al*., 2005. Microbiology. Sixth Edition; McGraw-Hill Medical Publishing.
7. Cappuccino, J. G. and N. Sherman. 2013. Microbiology: a laboratory manual. Tenth Edition; Pearson Education.

Course Code: ENG-1107 **Title:** Functional English **Credit Hrs:** 03

Contents: **Foundations of Functional English:** Vocabulary building (contextual usage, synonyms, antonyms and idiomatic expressions)**,** Communicative grammar (subject-verb-agreement, verb tenses, fragments, run-ons, modifiers, articles, word classes, etc.)**,** Word formation (affixation, compounding, clipping, back formation, etc.)**,** Sentence structure (simple, compound, complex and compound-complex)**,** Sound production and pronunciation**. Comprehension and Analysis:** Understanding purpose, audience and context**,** Contextual interpretation (tones, biases, stereotypes, assumptions, inferences, etc.)**,** Reading strategies (skimming, scanning, SQ4R, critical reading, etc.)**,** Active listening (overcoming listening barriers, focused listening, etc.)**. Effective communication:** Principles of communication (clarity, coherence, body, conciseness, courteousness, correctness, etc.)**,** Structuring documents (introduction, body, conclusion and formatting)**,** Inclusivity in communication (gender-neutral language, stereotypes, cross-culture communication, etc.)**,** Public speaking (overcoming stage fright, voice modulation and body language)**,** Presentation skills (organization content, visual aids and engaging the audience)**,** Information communication (

small talk, networking and conversational skills**),** Professional writing (business e-mails, memos, reports, formal letters, etc.).

Text and Recommended Books:

1. "Understanding and Using English Grammar" by Betty Schrampfer Azar.
2. "English Grammar in Use" by Raymond Murphy.
3. "The Blue Book of Grammar and Punctuation" by Jane Straus.
4. "English for Specific Purpose: A Learning-Centered Approach" by Tom Hutchinson and Alan Waters.
5. "Cambridge English for Job-hunting" by Colm Downes.
6. "Practical English Usage" by Michael Swan.
7. "Reading Literature and Writing Argument" by Missy James and Alan P. Merickel.
8. "Improving Reading: Strategies, Resources, and common Core Connections" by Jerry Jhons and Susan Lenski.
9. "Comprehension: A Paradigm for Cognition" by Walter Kintsch.
10. "Communication Skills for Business Professionals" by J.P. Verma and Meenakshi Raman.

Course Code: ISL-1112 **Title:** Islamic Studies **Credit Hrs:** 02

Contents: Introduction to Islam: Definition of Islam and its core beliefs, The Holy Quran (introduction, revelation, and compilation), Hadith and Sunnah (compilation, classification and significance), Key theological concept and themes (Tawhid, Prophethood, Akhairah etc.). **Sirah of the Holy Prophet (Peace Be Upon Him) as Uswa-i-Hasana:** Life and legacy of the Holy Prophet PBHU, Diverse roles of the Holy prophet PBHU (as an individual, educator, peace maker, leader etc.), **Islamic History and Civilization:** World before Islam, The Rashidun Caliphate and expansion of Islamic rule, Contribution of Muslim scientists and philosophers in shaping world civilization, **Islamic Jurisprudence (Fiqh):** Fundamental source of Islamic Jurisprudence, Pillars of Islam and their significance, Major School of Islamic Jurisprudence, Significance and principle of Ijtihad. **Family and Society in Islam:** Status and rights of women in Islamic teachings, Marriage, family and gender roles in Muslim society, Family structure and values in Muslim society. **Islam and the Modern World:** Relevance of Islam in the modern world (globalization, challenges and prospects), Islamophobia, interfaith dialogue and multiculturalism, Islamic viewpoint towards socio-culture and technological change.

Texts and Recommended books:

1. "The Five Pillars of Islam: a Journey through the Divine Acts of Worship" by Muhammad Mustafa Al-Azami.
2. "The Five Pillars of Islam: a Framework for Islamic Values and Character Building" Musharraf Hussain.
3. "Towards Understanding Islam" by Abul A' la Mawdudi.
4. "Islami Nazria e Hayat" by Khurshid Ahmad.
5. "An Introduction to Islamic Technology" by Jhon Renard.
6. "Islamic Civilization Foundations Belief & Principles" by Abul A' la Mawdudi.
7. "Women and Social Justice: An Islamic Paradigm" by Dr. Anis Ahmad.
8. "Islam: Its Meaning and Message" by Khurshid Ahmad.

Course Code: QTR-1124 Title: Quantitative Reasoning I Credit Hrs: 03

Course Outline: Numerical Literacy: Number system and basic athematic operations, Units and their conversions, dimensions, area, perimeter and volume, Rates, ratios, proportions and percentages, Types and sources of data, Measurements scales, Tabular and graphical presentation of data, Quantitative reasoning exercises using number knowledge. **Fundamental Mathematical Concepts:** Basics of geometry (lines, angles, circles, polygons, etc.)**,** Sets and their operations**,** Relations, functions, and their graphs, Exponents, factoring and simplifying algebraic expressions, Algebraic and graphical solutions of linear and quadratic equations and inequalities, Quantitative reasoning exercises using fundamental mathematical concepts. **Fundamental Statistical Concepts:** Population and sample, Measure of central tendency, dispersion and data interpretation, Rules of counting (multiplicative, permutation and combination), Basic probability theory, Quantitative reasoning exercises using fundamental statistical concepts.

Texts and Reference Books

1. "Quantitative Reasoning: Tools for Today's Informed Citizen" by Bernard L. Madison, Lynn and Arthur Steen.
2. "Quantitative Reasoning for the Information Age" by Bernard L. Madison and David M. Bressoud.
3. Fundamentals of Mathematics" by Wade Ellis.
4. "Quantitative Reasoning: Thinking in Numbers" by Eric Zaslow.

5. “Thinking Clearly with Data: A Guide to Quantitative Reasoning and Analysis” by Ethan Bueno de Mesquita and Anthony Fowler.
6. “Using and Understanding Mathematics: A Quantitative Reasoning Approach” by Bennet, J. O., Briggs, W. L., & Badalamenti, A.
7. “Discrete Mathematics and its Applications” by Kenneth H. Rosen.
8. “Statistics for Technology: A Course in Applied Statistics” by Chatfield, C.
9. “Statistics: Unlocking the Power of Data” by Robin H. Lock, Patti Frazer Lock, Kari Lock Morgan, and Eric F. Lock.

Course Code: ICT-1126 **Title:** Application of information and communication Technologies **Credit Hrs:** 03

Contents: Introduction to Information and Communication Technologies:

Components of Information and Communication Technologies (basics of hardware. Software, ICT platforms, networks, local and cloud data storage etc.).

Scope of information Communication technologies (use of ICT in education, business, governance, healthcare, digital media and entertainment, etc.), Emerging technologies and future trends. **Basic ICT Productivity Tools:** Effective use of popular search engines (e.g., Google, Bing, etc.) to explore World Wide Web, Formal communication tools and etiquettes (Gmail, Microsoft Outlook, etc.), Microsoft Office Suite (Word, Excel, PowerPoint), Google Workspace (Google Does, Sheets, Slides).

Dropbox (Cloud storage and file sharing), Google Drive (Cloud storage with Google Does integration) and Microsoft OneDrive (Cloud storage with Microsoft Office integration), Evernote (Note-taking and organization applications) and OneNote (Microsoft’s digital notebook for capturing and organizing ideas), Video conferencing (Google Meet, Microsoft Teams, Zoom, etc.), Social media applications (LinkedIn, Facebook, Instagram, etc.). **ICT in Education:** Working with learning management systems (Moodle, Canvas, Google Classrooms, etc.), Sources of online education courses (Coursera, edX, Udemy, Khan Academy, etc.), Interactive multimedia and virtual classrooms. **ICT in Health and Well-being:** Health and fitness tracking devices and applications (Google Fit, Samsung Health, Apple Health, Xiaomi Mi Band, Runkeeper, etc.), Telemedicine and online health consultations (OLADOC, Sehat Kahani, Marham, etc.). **ICT in Personal Finance and Shopping:** Online banking and financial management tools (JazzCash, Easypaisa, Zong PayMax, ILINK and MNET, Keenu Wallet, etc.), E-commerce platforms (Daraz. pk, Telemart, Shophive, etc.). **Digital Citizenship and Online Etiquette:** Digital identity and online reputation, Netiquette and respectful online communication, Cyberbullying and online harassment. **Ethical**

Consideration in Use of ICT Platforms and Tools: Intellectual property and copyrights issue, Ensuring originality in content creation by avoiding plagiarism and unauthorized use of information sources.

Content accuracy and integrity (ensuring that the content shared through ICT platforms is free from misinformation, fake news, and manipulation).

Texts and Recommended books:

1. Discovering Computers" by Vermaat, Shaffer and Freund.
2. "GO! With Microsoft Office" Series by Gaskin, Vargas and McLellan.
3. "Exploring Microsoft Office" Series by Grauer and Poastsy.
4. **"**Computing Essentials" by Morley and Parker.
5. "Technology in Action" by Evans, Martin, and Poatsy

Semester-II

Course Code: MATH-1201 **Title:** Calculus-II **Credit Hrs:** 03

Course Outline:

Techniques of integration: Integrals of elementary, hyperbolic, trigonometric, logarithmic and exponential functions. Integration by parts, substitution and partial fractions. Approximate integration. Improper integrals. Reduction Formulas, Gamma functions.

Applications of integrals: Area between curves, average value. Volumes. Arc length. Area of a surface of revolution. Applications to Economics, Physics, Engineering and Biology.

Infinite series: Sequences and series. Convergence and absolute convergence. Tests for convergence: divergence test, integral test, p-series test, comparison tests, alternating series test, ratio test, root test. Power series. Convergence of power series. Representation of functions as power series. Differentiation and integration of power series. Taylor and McLaurin series. Approximations by Taylor polynomials.

Conic section, parameterized curves and polar coordinates: Curves defined by parametric equations. Calculus with parametric curves: tangents, areas, arc length. Polar coordinates. Polar curves, tangents to polar curves. Area and arc length in polar coordinates.

Texts and Reference Books

1. H. Anton, *Calculus: A New Horizon*, 6th Edition, John Wiley and Sons, New York, 1999
2. J. Stewart, *Calculus*, 3rd Edition, Brooks/Cole, 1995.
3. G. B. Thomas and A. R. Finney, *Calculus*, 10th Edition, Addison Wesley, Reading, Ma, USA, 2002
4. M. Ryan, *Calculus, 2nd Edition*, For Dummies, 2016.
5. M. D. Spivak, *Calculus*, 4th Edition, Publish or Perish, 2008.

Course Code: MATH-1202 **Title:** Linear Algebra-I **Credit Hrs:** 03

Course Outline: System of Linear Equations, Row Reduction, Echelon and Reduced Forms, Vector Equations and the Matrix Equation, Solution of Linear Systems and Applications, **Matrix Operations**, Inversion and Characterizations of Invertible Matrices, Matrix Factorization, Application of Matrices and Linear Equations, Determinants and Their Properties, Cramer's Rule, Volume and Linear Transformations,
Fields, Vector Spaces, Subspaces, Linear Dependence and Independence, Linear Span of a Subset of a Vector Space, Bases and Dimensions, Rank and Nullity

Texts and Reference Books

1. D. C. Lay, *Linear Algebra and its Applications*, Dorling Kindersley Publishing, 2003.
2. S. J. Leon, *Linear Algebra with Applications*, 6th Edition, Prentice Hall, 2002.
3. G .E. Shilov, *Linear Algebra*, Dover Publication, Inc., New York, 1997.
4. D. G. Zill and M. R. Cullen, Advanced Engineering Mathematics, PWS, Publishing Company, Boston, 1996.
5. G. Strang, *Linear Algebra*, 5th Edition, Wellesley-Cambridge Press, 2016.

Course Code: ENG-1207 **Title:** Expository Writing **Credit Hrs:** 03

Contents: Self-Reflection, Introduction to the basics of the writing process, Introduction to the steps of essay writing, Students practice prewriting activities like brainstorming, listing, clustering and freewriting, Students practice, outlining of the essay, Discuss language learning experiences in English; produce a short essay describing language learning and writing experiences; revise writing based on feedback from peers,Students reflect on their learning process,Group discussion about learning styles based on the reading material provided to students, Introduction to personalized learning, Students practice goal setting and create a learning plan, Collaborate with peersto write a well- organized and concise list of guidelines that aregrammatically parallel; demonstrate fluency in English in group discussions and oral presentations; present ideas to the whole class in a team presentationusing English that is comprehensible and engaging, Introduction to the structure and significance of oral presentations, Class discussion about content selection and slide preparation for oral presentations,Peer review through a gallery walk , Critical Reading Skills, Introduce authentic reading (DAWN newspaper and non- specialist academic books/texts, Conduct classroom reading activities (using strategies skimming, scanning, SQW3R, previewing, annotating, detailed reading and note- taking) using standard tests (TOEFL and IELTS), Showing short documentaries to students on global environmental issues,Student-led brainstorming on local versus global issues,Teacher-led introduction to the unit assignment (using assignment sheet),Readings (or other input sources - video, social media) from local news on possible community issues, letters to editor and op-eds, Identify research problems, Facilitating students on developing research questions ingroups,Draft interview or survey questions forcommunity research(in English or L1), In-class role-plays ofinterviews with community members, Engaging students incritical reading and reflection on the issues found in different communities, In-class work on understanding interview information, how to present interview or survey information,Refining the research questions, designing a detailed research plan in groups, dividing the tasks and deciding the timeline for the completion of the project, Exposure to interview questions and interviewing techniques to developan in-depth understanding of the issues, Teacher feedback on outline of report (globally to entire class and individually to groups as needed),Revisions to oral report in groups,Engaging students in

individual structured reflective writing based on their experience of working on the project,Sharing their reflective writing to learn about each other's points of view, Think-pair-share the findings (group similar issues), Individual writing of reflection on the community engagement project and their role in the group,Brainstorm using creativity for dissemination - cartoons, advertisements for university magazine or beyond, creating posts for FB,Summarizing/ converting the report to a letter to the editor to highlight the problems explored and their possible solutions,Teacher-directed instruction on genres (types) of writing focusing on letter-writing, Model-practice- reflect: Introduce types of letters comparing the use of formal and informal vocabulary and phrases in each type,Introduce the format and purpose of the letter-to-editor explaining with the help of an actual letter from a local newspaper, Group reading of sample letters-to- editor selecting ones that deal with issues familiar to the students, Ask students to identify features of the "letter to the editor" genre, Assign different letters to the students and ask them to read critically to dentify strength andweaknesses of model letters, Provide meaningfulpeer feedback on outlines and drafts, Invite a guest lecturer (local newspaper editor or faculty from journalism) to talk about what issues are currently raised in letters-to-editors and what are editors' criteria to accept letters for publication,Work in groups to continue reviewing letter samples, analyzing the structure of letters, Each group identifies an issue they want to write about and give a brief oral presentation to the class, Draft a solution-focused letter using supporting evidence, Demonstrate problem- solving skills through letter writing, Ask students to submit the first draft of letters (to the teacher and peer- review group), In-class peer review of drafts using a checklist focusing on content and structure, Differentiate among revision, proofreading and evaluation (as sub stages of finalizing documents), Discuss critically the draft-letter and implement the 'revision' phase of writing, Reading of (DAWN) newspaper and sharing important letters (to editors) on local issues, Explicit instruction (paragraph structure, syntax, diction, grammar, and mechanics), Classroom discussion/debrief of activity, Discuss critically and finalize the draft- letter as the last phase of writing

Reference Books

1. Community Engagement Toolkit for Planning (2017) Guiding principles [pp7-24] Developing Community Engagement Plan [pp13-29].

https://examples.yourdictionary.com/examples-of-good-and-badresearchquestions.html

2. Examine Applicable Strategies Accessed at: https://opentextbc.ca/s tudentsuccess/chapter/ Hall, Hellen (2012) Reverse Outlines Reverse Outlines: Take A part Your Paper to Put it Back Together Right. Accessed at:

https://www.semantic scholar.org/paper/Reverse-Outline-s- Reverse-Outlines-%3A-Take-Apart-to- Hall/c0373e42616395 ea9edf5d5bd5cbe6eb 1bb923e2

3. Learning Preferences and Strengths Accessed at: https://opentextbc.ca/studentsuccess/chapter/learning-preferences- and-strengths/
4. Organizing an Essay Accessed at:/https://courses.lumenl earning.com/englishco mp1v2xmaster/chapter/organizing-an-essay/
5. Planning the Presentation Accessed at: https://opentextbc.ca/student success/chapter/ planning-the- presentation/ Warburton, N. (2020). The basics of essay writing. Routledge

Course Code: PHY-1218 **Title:** Physics-I **Credit Hrs:** 03

Contents: Work and Energy, Impulse and Momentum, Circular Motion, Mass and Energy, Collisions, Center of Mass, Moment, Center of Gravity, Couples, Angular Motion, Kinetic Energy of Rotation, Moment of Inertia, Parallel Axis Theorem, Momentum, Angular Momentum and Energy, Elastic Restoring Forces, Circle of Reference, Harmonic Motion

Density, Pressure in a Fluid, Pressure Gauges, Pumps, Archimedes' Principle, Forces against a Dam, Surface Tension, Pressure Difference across a Surface Film, Contact Angle and Capillarity

Equation of Continuity, Bernoulli's Equation, Applications of Bernoulli's Equation, Viscosity, Poiseuilles's Law, Stokes' Law, Reynolds Number

Waves and Mathematical Description, Speed of a Transverse Wave and Longitudinal Wave, Adiabatic Character of a Longitudinal Wave, Water Waves, Superposition and Standing Waves, Longitudinal Standing Waves, Vibrations of Organ Pipes, Vibrations of Rods and Plates, Interference of Longitudinal Waves, Resonance, Sound Waves, Doppler Effect, Radiation from a Piston, Applications of Acoustic Phenomena

Texts and Recommended books:

1. F. W. Sears, M. W. Zemansky and H. D. Young, *University Physics*, 3th Edition, 1963.
2. R. A. Serway and J. W. Jewett, Principles of Physics, 5th Edition, Brooks Cole, 2012.
3. H. C. Ohanian and J. T. Market, *Physics for Engineers and Scientists*, 3rd Edition, W. W. Norton & Company, 2006.
4. R. D. Knight, *Physics for Scientists and Engineers*, 2th Edition, Addison-Wesley, 2007

Course Code: QTR-1224 **Title:** Quantitative Reasoning II **Credit Hrs:** 03

Course Outline: Logic, Logical and Critical Reasoning: Introduction and importance of logic, Inductive, deductive and abductive approaches of reasoning, Propositions, arguments (valid; invalid), logical connectives, truth tables and propositional equivalences, Logical fallacies, Venn Diagrams, Predicates and quantifiers, Quantitative reasoning exercises using logical reasoning concepts and techniques. **Mathematical Modeling and Analysis:** Introduction to deterministic models, Use of linear functions for modeling in real-world situations, Modeling with the system of linear equations and their solutions, Elementary introduction to derivatives in mathematical modeling, Linear and exponential growth and decay model, Quantitative reasoning exercises using mathematical modeling. **Statistical Modeling and Analysis:** Introduction to probabilistic models, Bivariate analysis, scatter plots, Simple linear regression model and correlation analysis, Basics of estimation and confidence interval, Testing of hypothesis (z-test; t-test), Statistical inference in decision making, Quantitative reasoning exercises using statistical modeling.

Texts and Reference Books

1. Using and Understanding Mathematics: A Quantitative Reasoning Approach by Bennett, O., Briggs, W. L., & Badalamenti, A.
2. Discrete Mathematics and its Applications by Kenneth H. Rosen.

3. "Discrete Mathematics with Applications" by Susanna S. Epp.
4. "Applied Mathematics for Business, Economics and Social Sciences" by Frank S Budnick.
5. "Elementary Statistics: A Step by Step Approach" by Allan Bluman.
6. Introductory Statistics" by Prem S. Mann.
7. "Applied Statistical Modeling" by Salvatore Babones.
8. "Barrons SAT" by Sharvon Weiner Green, M.A and Ira K. Wolf.

Course Code: ICP-1227 **Title:** Ideology and constitution of Pakistan **Credit Hrs:** 02

Contents: Introduction to the Ideology of Pakistan: Definition and significance of ideology. Historical context of the creation of Pakistan (with emphasis on socio-political, religious, and cultural dynamics of British India between 1857 till 1947).

Contributions of founding fathers of Pakistan in the freedom movement including but not limited to Allama Muhammad Iqbal, Muhammad Ali Jinnah., etc, Contributions of women and students in the freedom movement for separate homeland for Muslims of British India.

Two-Nation Theory:

Evolution of the Two-Nation Theory (Urdu-Hindi controversy, Partition of Bengal, Simla Deputation 1906, Allama Iqbal's Presidential Address 1930, Congress Ministries 1937 Lahore Resolution 1940), Role of communalism and religious differences. **Introduction to the Constitution of Pakistan:** Definition and importance of a constitution, Ideological factors that shaped the Constitution(s) of Pakistan (Objectives Resolution 1949), Overview of constitutional developments in Pakistan. **Constitution and State Structure:** Structure of Government (executive, legislature, and judiciary), Distribution of powers between federal and provincial governments, 18th Amendment and its impact on federalism. **Fundamental Rights, Principles of Policy and Responsibilities:** Overview of fundamental rights guaranteed to citizens by Constitution of Pakistan 1973 (Articles 8-28), Overview of Principles of Policy (Articles 29-40), Responsibilities of the Pakistani citizens (Article 5). **Constitutional Amendments:** Procedures for amending the Constitution, Notable constitutional amendments and their implications.

Texts and Recommended books:

1. "The Idea of Pakistan" by Stephen P. Cohen.
2. "Ideology of Pakistan" by Javed Iqbal.
3. "The Structure for Pakistan" by I.H. Qureshi.
4. "Pakistan the Formative Phase" by Khalid Bin Sayeed.
5. "Pakistan: Political Roots and Development" by Safdar Mahmood.
6. "Ideology of Pakistan" by Sharif-ul-Mujahid.

7. “The Struggle for Pakistan: A Study Homeland and Global Politics” by Ayesha Jalal.
8. “Jinnah, Pakistan and Islamic Identity: The Search for Saladin” by Akbar S. Ahmed.
9. “The Making of Pakistan: A Study in Nationalism” by K.K. Aziz.
10. “Pakistan: A New History” by Ian Talbot.
11. “Pakistan in the twentieth Century: A Political History” by Lawrence Ziring.
12. “The Constitution of Pakistan 1973”. Original.
13. “Constitutional and Political Development of Pakistan” by Hamid Khan.
14. “The Parliament of Pakistan” by Mahboob Hussain.
15. “Constitutional Development in Pakistan” by G/W. Choudhury.
16. “Constitutional-Making in Pakistan: The Dynamics of Political Order” by G.W. Choudhury.

Semester-III

Course Code: MATH-2301 **Title:** Calculus-III **Credit Hrs:** 03

Course Outline:

Multivariable functions and partial derivatives: Functions of several variables. Limits and Continuity. Partial derivatives, Composition and chain rule. Directional derivatives and the gradient vector. Implicit function theorem for several variables. Maximum and minimum values. Optimization problems. Lagrange Multipliers.

Vectors and analytic geometry in space: Coordinate system. Rectangular, cylindrical and spherical coordinates. The dot product, the cross product. Equations of lines and planes. Quadric surfaces.

Vector-valued functions: Vector-valued functions and space curves. Derivatives and integrals of vector valued functions. Arc length. Curvature, normal and binormal vectors.

Multiple integrals: Double integrals over rectangular domains and iterated integrals. Non-rectangular domains. Double integrals in polar coordinates. Triple integrals in rectangular, cylindrical and spherical coordinates. Applications of double and triple integrals. Change of variables in multiple integrals.

Vector calculus: Vector fields. Line integrals. Green's theorem. Curl and divergence. Surface integrals over scalar and vector fields. Divergence theorem. Stokes' theorem.

Texts and Reference Books

1. H. Anton, *Calculus: A New Horizon*, 6th Edition, John Wiley and Sons, New York, 1999.
2. J. Stewart, *Calculus*, 3rd Edition, Brooks/Cole, 1995.
3. G. B. Thomas and A.R. Finney, *Calculus*, 10th Edition, Addison Wesley, Reading, Ma, USA, 2002.
4. M. Ryan, *Calculus, 2nd Edition*, For Dummies, 2016.
5. Dr. R. Larson, *Calculus*, 9th Edition, Brooks/Cole, 2009.

Course Code: MATH-2302 **Title:** Linear Algebra-II **Credit Hrs:** 02

Course Outline: Linear Transformations and the Matrix of Linear Transformations, Eigenvectors and eigenvalues, Diagonalization, Complex Eigenvalues, Inner Product Spaces, Length and Orthogonality, Orthogonal Sets and Orthogonal Projections, Gram-Schmidt Process, Symmetric Matrices, Diagonalization of Symmetric Matrices, Quadratic Forms

Texts and Reference Books

1. D. C. Lay, *Linear Algebra and its Applications*, Dorling Kindersley Publishing, Inc., 2003.
2. S. J. Axler, *Linear Algebra*, Done Right, Springer-Verlag, 1996.
3. W. K. Nicholson, *Elementary linear algebra with applications*, 2nd Edition, PWS Publishing Company, 1994.
4. D.G. Zill and M. R. Cullen, *Advanced Engineering Mathematics*, PWS, Publishing Company, Boston, 1996.
5. M. J. Sterling, *Algebra II*, 2nd Edition, For Dummies, 2014.

Course Code: MATH-2303 **Title:** Mathematical Statistics I **Credit Hrs:** 03

Course Outline: The postulates of probability, Some elementary theorems, Addition and multiplication rules, Baye's rule and future Baye's theorem, Random variables and probability functions, Uniform, Bernoulli and Binomial distribution, Hypergeometric and geometric distribution, Negative binomial and Poisson distribution, Uniform and exponential distribution, Gamma and beta distributions, Normal distribution, Moments and moment generating functions Moments of binomial, hypergeometric, Poisson, gamma, beta and normal distributions

Texts and Reference Books

1. M. H. De-Groot and M. J. Schervish, *Probability and Statistics*, 3rd Edition, Addison Wesley, 2002.
2. A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd Edition, Mc-Graw Hill, 1991.
3. T. Sincich, *Statistics by Examples*, Dellen Publishing Company, 1990.
4. A. S. Hirahi. *A Course in Mathematical Statistics*, 4th Edition, Ilmi Kitab Khana, Lahore, Pakistan, 2012.
5. S. M. Chaudhry and S. Kamal, *Introduction to Statistical Theory Part-II*, Ilmi Kitab Khana, Lahore, Pakistan, 2012.

Course Code: MATH-2304 **Title:** Mechanics-I **Credit Hrs:** 03

Course Outline:

Forces: Fundamental concepts and principles, Newtonian Mechanics, Inertial-non-inertial frames, Resultant of several concurrent forces, The parallelogram law of forces, Resolution of a forces, triangle of forces, Lamy's theorem, polygon of forces, Conditions of equilibrium for a particle, External and internal forces, principle of transmissibility, Resultant of like and unlike parallel forces, Moment of forces about a point, Varigon's theorem, Moment of a couple, equivalent couples, composition of couples, Reduction of coplanar forces to a force or a couple

Friction: Dry friction and fluid friction, Laws of dry friction, coefficients of friction, angle of friction, Equilibrium of a particle on a rough inclined plane, Particle on a rough inclined plane acted on by an external force, Conditions for sliding or titling

Virtual Work: Principle of virtual work, Problems involving tensions and thrust.

Texts and Reference Books:

1. A. Bedford and W. Fowler, *Dynamics Engineering Mechanics*, Addision-Wesley, Reading, USA.
2. T. L. Chow, *Classical Mechanics*, John Wiley and Sons, New York, 1995.
3. H. Goldstein, *Classical Mechanics*, 2nd Edition, Addison Wesley, Reading, Ma, USA, 1980.
4. J. B. Marion, *Classical Dynamics of Particles and Fields*, 2nd Edition, Academic Press, New York, 1970.
5. J. R. Taylor, *Classical Mechanics*, University Science Books, 2005.

Course Code: HRM-2309 **Title:** HR Management **Credit Hrs:** 02

Course Contents:

Introduction to HRM, Human Resource Planning, Job Design and Analysis, Recruitment & Selection, Motivation & Reward System, Career Planning & Development, Training & Development, Performance Appraisal, Compensation Management & Employee Relation, Employee Health and Safety.

Text and Reference books:

1. R. A. Noe, P. M. Wright, J. R. Hollenbeck and B. Gerhart , *Human Resource Management* , 10th edition, McGraw-Hill Education, 2016.
2. R. L. Mathis and P. Meglich, *Human Resource Management*, 15th edition, South-Western College Pub, 2016.
3. B. Wolfe, *The Little Black Book of Human Resources Management*, Create Space Independent Publishing Platform, 2015.
4. J. H. Jackson, *Human Resource Management*, 14th Edition, South-Western College Pub, 2013.

Course Code: PSY-2319 **Title**: Introduction to Sociology **Credit Hrs:** 03

Contents: Introduction, Scope and Subject Matter, Sociology as a Science, Historical back ground, Basic Concepts, Social Interaction: Levels of Social Interaction, Social Groups, Culture, Norms and Social Sanctions, Socialization & Personality, Deviance and Social Control, Collective Behavior

Texts and Recommended books:

1. Anderson, Margaret and Howard F. Taylor, *Sociology the Essentials*, Australia, Wadsworth, 2001.
2. Brown Ken, *Sociology*, UK: Polity Press, 2004.
3. Gidden, Anthony, *Introduction to Sociology*, UK: Polity Press, 2002.
4. J. Macionis John, *Sociology*, 10th Edition New Jersey: Prentice-Hall, 2006.
5. L.Tischler Henry, *Introduction to Sociology*, 7th ed. New York: The Harcourt Press, 2002.
6. N. Frank Magill, *International Encyclopedia of Sociology*, U.S.A: Fitzroy Dearborn Publishers, 2003.
7. J.Macionis John, *Sociology*, 10th ed. South Asia: Pearson Education, 2005.
8. R.Kerbo Harold, *Sociology: Social Structure and Social Conflict*, Macmillan Pub. Co., 1989.
9. Koening Samuel, *Sociology: An Introduction to the Science of Society*, Barnes and Nobel, 1957.

Course Code: CCE-2325 Title: Civic and Community Engagement **Credit Hrs: 02**

Course Contents: Civics and Citizenship: Concepts of civics, citizenship and civic engagement, Foundations of modern society and citizenship, Types of citizenship: active, participatory, digital, etc. **State, Government and Civil Society:** Structure and functions of government in Pakistan, The relationship between democracy and civil society, Right to vote and importance of political participation and representation. **Rights and Responsibilities:** Overview of fundamental rights and liberties of citizens under Constitution of Pakistan 1973, Civic responsibilities and duties, Ethical

considerations in civic engagement (accountability, non-violence, peaceful dialogue, civility, etc.)**. Community Engagement:** Concept, nature and characteristics of community, Community development and social cohesion, Approaches to effective community engagement, Case studies of successful community driven initiatives. **Advocacy and Activism:** Public discourse and public opinion, Role of advocacy in addressing social issues, Social action movements. **Digital Citizenship and Technology:** The use of digital platforms for civic engagement, Cyber ethics and responsible use of social media, Digital divides and disparities (access, usage, socioeconomic, geographic, etc.) and their impacts on citizenship, **Diversity, Inclusion and Social Justice,** Understanding diversity in society (ethnic, cultural, economic, political etc.), Youth, women and minorities' engagement in social development, Addressing social inequalities and injustices in Pakistan, Promoting inclusive citizenship and equal rights for societal harmony and peaceful coexistence.

Texts and Reference Books:

1. "Civics Today: Citizenship, Economics, & You" by McGraw-Hill Education.
2. "Citizenship in Diverse Societies" by Will Kymlicka and Wayne Norman.
3. "Engaging Youth in Civic Life" by James Youniss and Peter Levine.
4. "Digital Citizenship on Action: Empowering Student to Engage in Online Communities" by Kristen Mattsen.
5. "Globalization and Citizenship: In the Pursuit of a Cosmopolitan Education" by Graham Pike and David Selby.
6. "Community Engagement: Principles, Strategies, and Practices" by Becky J. Feldpausch and Susan M. Omilian.
7. "Creating Social Change: A Blueprint for a better world" by Matthew Clarke and Marie-Monique Steckel.

Semester-IV

Course Code: ARA-2401 **Title:** Arabic **Credit Hrs:** 02

Course Contents: Arabic to Urdu Translation, Urdu to Arabic Translation, Grammar, Comprehension.

Text and Reference Books: Lisaan Ul Arabi, AIOU, Isalamabad**.**

Course Code: MATH-2402 **Title:** Differential Equation I **Credit Hrs:** 02

Course Outline:

Preliminaries: Introduction (Formulation and classifications of differential equations), existence and uniqueness of solutions, introduction of initial value and boundary value problems

First order ordinary differential equations: Basic concepts, Separable variables, Exact Equations, Homogeneous Equations, Linear equations, integrating factors. Some nonlinear first order equations with known solution, differential equations of Bernoulli and Ricaati type, Clairaut equation, modeling with first-order ODEs,

Second and higher order Linear differential equations: Initial value and boundary value problems, Homogeneous and non-homogeneous equations, Superposition principle, homogeneous equations with constant coefficients, Linear independence and Wronskian, Nonhomogeneous equations, undetermined coefficients method, variation of parameters, Cauchy-Euler equation, Modeling.

Texts and Reference Books

1. W. E. Boyce and R. de Prima, *Elementary Differential Equations*, 9th Edition, Wiley, 2008.
2. E. Kreyszig, *Advanced Engineering Mathematics*, 10th Edition, Wiley, 2011.
3. D. G. Zill, *Differential Equations with Boundary-Value Problems*, 8th Edition, Brooks Cole, 2012.
4. S. J. Farlows, *An Introduction to Differential Equations and Their Applications*, Dover Publications, 2006.
5. M. E. Taylor, *An Introduction to Differential Equations*, 14th Edition, American Mathematical Society, 2011.

Course Code: MATH-2403 **Title:** Mathematical Statistics II **Credit Hrs:** 03

Course Outline: Distribution function technique, Transformation technique: One variable, several variables, Moment-generating function technique, The distribution of the mean, The distribution of the mean: Finite populations, The Chi-Square distribution., The t distribution, The F distribution, Regression and Correlation, Linear regression, The methods of least squares, Normal regression analysis, Normal correlation analysis, Multiple linear regression, Multiple linear regression (matrix notation)

Texts and Reference Books

1. M. H. De-Groot and M. J. Schervish, *Probability and Statistics*, 3rd Edition, Addison Wesley, 2002.
2. A. Papoulis, *Probability, Random Variables, and Stochastic Processes*, 3rd Edition, Mc-Graw Hill, 1991.
3. T. Sincich, *Statistics by Examples*, Dellen Publishing Company, 1990.
4. A. S. Hirahi. *A Course in Mathematical Statistics*, 4th Edition, Ilmi Kitab Khana, Lahore, Pakistan, 2012.
5. S. M. Chaudhry and S. Kamal. *Introduction to Statistical Theory Part-II***,** Ilmi Kitab Khana, Lahore, Pakistan, 2012.

Course Code: MAT-2404 Title: Mechanics-II Credit Hrs: 03

Course Outline:

Kinematics: Rectilinear motion of particles. Uniform rectilinear motion, uniformly accelerated rectilinear motion. Curvilinear motion of particle, rectangular components of velocity and acceleration. Tangential and normal components. Radial and transverse components. Projectile motion. Kinetics: Work, power, kinetic energy, Conversation Laws (conservative force fields. Conservation of energy Conservation of linear and angular momentum), impulse, torque. Non-

conservative forces. Simple Harmonic Motion: The simple harmonic oscillator, period, frequency. Resonance and energy. The damped harmonic oscillator, over damped, critically damped and under damped. Motion, forces and vibrations. Central Forces and Planetary Motion: Central force fields, Properties of Central Force Field, Equations of motion, potential energy, orbits. Kepler's law of planetary motion. Apsides and apsidal angles for nearly circular orbits. Motion in an inverse square field.

Planer Motion of Rigid Bodies: Introduction to rigid and elastic bodies, degree of freedom, translations, rotations, instantaneous axis and center of rotation, Rotation of a rigid body about a fixed axis, moments and products of inertia. Parallel and perpendicular axis theorem.

Texts and Reference Books

1. A. Bedford and W. Fowler, *Dynamics Engineering Mechanics*, Addision-Wesley, Reading, USA.
2. T. L. Chow, *Classical Mechanics*, John Wiley and Sons, New York, 1995.
3. H. Goldstein, *Classical Mechanics*, 2nd Edition, Addison Wesley, Reading, Ma, USA, 1980.
4. J. B. Marion, *Classical Dynamics of Particles and Fields*, 2nd Edition, Academic Press, New York, 1970.
5. J. R. Taylor, *Classical Mechanics*, Null Edition, University Science Books, 2005.

Course Code: MATH-2405 Title: Metric Spaces Credit Hrs: 02

Course Outline: Preliminary Concepts, Definition and Examples of Metric Spaces, Open and Closed Spheres and Sets, Convergent Sequences, Cauchy Sequences, Cantor's Intersection Theorem, Complete Metric Spaces, Dense and Nowhere Dense Subsets, Continuous and Uniform Continuous Functions and Their Properties,

Texts and Reference Books

1. C. W. Patty, *Foundation of Topology*, 2nd Edition, The Jones and Bartlett Publishers, 2009.
2. J. Dugundji, *Topology*, W.M. C. Brown Publisher, 1990.
3. E. Kreyszig, *Introductory Fundamental Analysis with Applications*, John Wiley and Sons, 1978.
4. M. O. Searcoid, *Metric Spaces*, 2007 Edition, Springer, 2006.
5. P. K. Jain, *Metric Spaces*, 2nd Edition, Alpha Science Intl Ltd, 2004.

Course Code: MATH-2406 Title: Business Mathematics Credit Hrs: 03

Contents: Basic mathematical concepts and their application in various business real numbers, Linear equations and their applications, System of linear equations, Functions (linear functions, quadratic and polynomial functions, exponential and logarithmic functions), Matrix algebra, Differentiation and Integration, Mathematics of finance, Ratios, Proportions and Percentages, Principle of simple Interest, Principle of compound Interest, Annuities, Loans and Mortgages, Investment decisions, Discrete and continuous variables, Linear equations and inequalities and

graphs, Progression of Sequence and Series and their applications in business, Permutations, Combination and their applications in business and finance.

Texts and Recommended books:

1. Frank S. *Budnick: Applied Mathematics for Business and Economics and Social sciences*.
2. E. K. Bowen and G. D. Prichett, *Mathematics with applications in Management and Economics*, 6th Edition, McGraw-Hill Inc, 1986.
3. G. Clendenen and S. A. Salzman, *Business Mathematics*, 13th Edition, Pearson, 2014.
4. C. D. Miller, G. Clendenen and S. A. Salzman, *Business Mathematics*, 12th Edition, Pearson, 2011.

Course Code: ETR-2408 **Title:** Entrepreneurship **Credit Hrs:** 02

Course Contents:

Introduction to Entrepreneurship: Definition and concept of entrepreneurship, Why to become an entrepreneur? Entrepreneurial process, Role of entrepreneurship in economic development. **Entrepreneurial Skills:** Characteristics and qualities of successful entrepreneurs (including stories of successes and failures), Areas of essential entrepreneurial skill and ability such as creative and critical thinking, innovation and risk taking abilities etc. **Opportunity Recognition and Idea Generation:** Opportunity identification, evaluation and exploitation, Innovative idea generation techniques for entrepreneurial ventures. **Marketing and Sales:** Target market identification and segmentation, Four P's of Marketing, Developing a marketing strategy, Branding. **Financial Literacy:** Basic concepts of income, savings and investments, Basic concepts of assets, liabilities and equity, Basic concepts of revenue and expenses, Overview of cash-flows, Overview of banking products including Islamic modes of financing, Sources of funding for startups (angle financing, debt financing, equity financing etc.). **Team Building for Startups:** Characteristics and features of effective teams, Team building and effective leadership for startups. **Regularity Requirements to Establish Enterprises in Pakistan:** Types of enterprises (e.g., sole proprietorship; partnership; private limited companies etc.), Intellectual property rights and protection, Regulatory requirements to register an enterprise in Pakistan, with special emphasis on export firms, Taxation and financial reporting obligation.

Recommended Books:

1. "Entrepreneurship: Successfully Launching new Ventures" by Bruce R. Barringer and R. Duane Ireland.
2. "Entrepreneurship: Theory, Process, and Practice" by Donald F. Kuratko.

3. "New Venture Creation: Entrepreneurship for the 21st Century" by Jeffry A. Timmons, Stephen Spinelli Jr., and Rob Adams.
4. "Entrepreneurship: A Real-World Approach" by Rhonda Abrams.
5. "The Lean startup: How Today's Entrepreneurs Use Continuous Innovation to Create Radically Successful Businesses" by Eric Ries.

Course Code: IR-2410 Title: Introduction to International relations Credit Hrs: 03

Course Contents: Meaning, Definition, Nature, and Scope of International Relations, Evolution and Development of International Relations, Significance of International Relations, Concept of Nation State, Historical context of contemporary IR, International Relations Theories, International System and Sub-Systems, State and Non-state Actors, IO, IL and NGO, The Concept of War and Peace in International Relations, The Role of Economics in International Relations(IPE), Human Rights in International Relations, Latest/emerging concepts and trends(environment, global health), National Interest, Foreign Policy and Diplomacy, Power and Balance of Power, Regionalism and Globalization.

Texts and Recommended books

1. Karen A. Mingst, and Ivan M. Arreguín-Toft, Essentials of International Relations (London: Oxford Publishers, 2016)
2. Richard Devetak and Jim George, An Introduction to International Relations (Cambridge: Cambridge University Press, 2017)
3. Theodore Columbus, Introduction to International Relations: Power and Justice (New Delhi: Prentice Hall, 1992)
4. Joshua Goldstine, International Relation (Washington DC: Pearson Education, 2003)
5. Mark R. Amstutz, International Conflict and Cooperation: An Introduction to World Politics (Chicago: Brown & Benchmark, 1995)
6. Martin Griffiths, and Terry O'. Callaghan, International Relations: The Key Concepts (London, Rutledge, 2003)
7. Paul Wilkinson, International Relations: A Very Short Introduction (Oxford: Oxford University Press, 2007)

Course Code: OB-2416 Title: Organizational Behavior Credit Hrs: 03

Course Contents:

Introduction to HRM, Human Resource Planning, Job Design and Analysis, Recruitment & Selection, Motivation & Reward System, Career Planning & Development, Training & Development, Performance Appraisal, Compensation Management & Employee Relation, Employee Health and Safety.

Texts and Recommended books:

5. Garry Dessler, *Human Resource Management* .
6. Garry Dessler, *Human Resource Management.*
7. Dale S. Beach, *Personnel The Management of people at work* .
8. Holdin, *Human Resource Management* .

Course Code: **PSY-2419 Title: Educational Psychology Credit Hrs:** 03

Course Contents: Introduction, Nature and Application of Psychology with Special Reference to Pakistan, Historical Background (A Brief Survey), Methods of Psychological Study, Biological Basis of Behavior, Vision, Audition, Perception, Attention, Motives, Emotions, Learning, Memory, Tools of Thinking, Individual Differences

Texts and Recommended books:

1. R. C. Atkinson and E. E. Smith *Introduction to psychology*, Harcourt Brace College Publishers, 2000.

2. L.D. Fernald and P.S. Fernald, *Introduction to psychology*, USA: W M C Brown Publishers, 2005.
3. W. E. Glassman, *Approaches to psychology*, Open University Press, 2000.
4. N. Hayes *Foundation of psychology*, 3rd Edition, Thomson Learning, 2000.
5. B. B. Lahey, *Psychology: An introduction*, 8th Edition, Mc Graw Hill Companies, Inc. (2004).

Semester-V

Course Code: MATH-3501 Title: Real Analysis-I Credit Hrs: 03

Course Outline: Supremum and Infimum, Completeness Properties of the Real Numbers, Limits and Continuity, Properties of Continuous Functions on Closed Bounded Intervals, Derivatives in One Variable, The Mean Value Theorem, Sequences and Series of Functions, Power Series, Pointwise and Uniform Convergence, Riemann Integrations

Texts and Reference Books

1. H. L. Royden, *Real Analysis*, Mc-Millan Publishing Company, Inc., New York, 1968.
2. W. Rudin, *Principles of Real Analysis,* Mc-Graw Hill, 1995.
3. R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis*, 3rd Edition, John Wiley and Sons, 1999.
4. R. L. Brabenec, *Introduction to Real Analysis*, PWS Publishing Company, 1994.
5. E. D. Gaughan, *Introduction to Analysis*, 5th Edition, Brooks/Cole, 1997.

Course Code: MATH-3502 Title: Differential Equations II Credit Hrs: 03

Course Outline:

Review of first order linear and nonlinear odes and higher order linear odes and their solution techniques. **Basic theory** of systems of first order linear equations, Homogeneous linear system with constant coefficients, Non homogeneous linear system, **Series Solution and its Limitations,** The Frobenius Method, **Sturm-Liouville** (S-L) System and Boundary-Value Problems, Solution of the Bessel, The Hypergeometric, The Legendre and the Hermite Equations, Properties of the Bessel, the Legendre and the Hermite Functions

Texts and Reference Books

1. E. A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations,* Mc-Graw Hill, New York,Toronto and London, 1955.
2. W. E. Boyce and R. de Prima, *Elementary Differential Equations*, 9th Edition, Wiley, 2008.
3. P. Hartmen, *Ordinary Differential Equations,* John Wesley and Sons, New York,1964.
4. D. G. Zill and M. R. Cullen, *Differential Equations with Boundary-Value Problems*, 3rd Edition, PWS Publishing Company, 1997.
5. V. I. Arnold and R. Cooke, *Ordinary Differential Equations,* 2006 Edition, Springer, 2006.

Course Code: MATH-**3503** **Title: Differential Geometry-I** **Credit Hrs: 03**

Course Outline: Historical Background, Motivation and Applications, Index Notation and Summation Convention, Space Curves, The Tangent Vector Field, Reparametrization; Arc Length, Curvature, Principal Normal, Binormal, Torsion, The Osculating, The Normal and the Rectifying Planes, The Frenet-Serret Theorem, Spherical Images, Sphere Curves, Spherical Contacts, Fundamental Theorem of Space Curves, Line Integrals and Green's Theorem, Local Surface Theory, Coordinate Transformations, The Tangent and the Normal Planes, Parametric Curves, The First Fundamental Form and the Metric Tensor, Normal and Geodesic Curvatures, Gauss's Formulae, Christoffel Symbols of First and Second Kinds, Parallel Vector Fields Along a Curve and Parallelism, The Second Fundamental Form and the Weingarten Map, Principal, Gaussian, Mean and Normal Curvatures, DupinIndicatrices, Conjugate and Asymptotic Directions, Isometries and the Fundamental Theorem of Surfaces

Texts and Reference Books

1. M. P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, 1976.
2. A. Goetz, *Introduction to Differential Geometry*, Addison Wesley, 1970.
3. R. S. Millman and G. D. Parker, *Elements of Differential Geometry*, Prentice Hall, 1977.
4. A. N. Pressley, *Elementary differential geometry*, 2nd Edition, Springer-Verlag, 2012.
5. D. J. Struik, *Lectures on Classical Differential Geometry*, Addison Wesley, Massachusetts, 1977.

Course Code: MATH-3504 Title: Analytical Mechanics **Credit Hrs:** 03

Course Outline: Kinematics of Particles, Motion in Resisting Medium, Variable Mass Problem, Euler's Theorem and Chasles' theorem, **Moving Coordinate System:** Motion Relative to the Rotating Earth, **The Motion of a System of Particles**: Work, Power, Energy, center of mass, Generalized Coordinates, Lagrange's and Hamilton's Equations, Hamiltonian Principle, Simple Applications, Properties of a Rigid Body, Motion under No Forces, **Motion of Rigid Bodies in Three Dimensions:** General motion of rigid bodies in space. The momental ellipsoid and equimomental systems. Angular momentum vector and rotational kinetic energy. Principal axes and principal moments of inertia. Determination of 16 principal axes by diagonalizing the inertia matrix. **Euler Equations of Motion of a Rigid Body:** Force free motion. Free rotation of a rigid body with an axis of symmetry. Free rotation of a rigid body with three different principal moments. The Eulerian angles, angular velocity and kinetic energy in terms of Euler angles. Motion of a spinning top and gyroscopes-steady precession, sleeping top.

Texts and Reference Books

1. G. R Fowles, G.L. Cassiday, *Analytical Mechanics*, 7th Edition, Thomson Brook Cole, 2005.
2. B. Jafferson, T. Beasdsworth, *Further Mechanics*, Oxford University Press, 2001.
3. Louis N. Hand, *Analytical Mechanics*, 1st Edition, Cambridge University Press, 1998.
4. Dr. C. Helrich, *Analytical Mechanics*, 1st Edition, Springer, 2016.
5. J . S. Torook, *Analytical Mechanics*, 1st Edition, Wiley-Interscience, 1999.

Course Code: MATH-**3505** **Title: Topology** **Credit Hrs:** 03

Course Outline: Overview of Metric Spaces, Convergence, and Continuity in Metric Spaces, Topological Spaces, Sub-Spaces, Closed Sets, Closures and Interiors of Sets, Boundary of a Set, Limit Point, Bases and Sub-bases, Neighborhood Base, First and Second Countable Spaces, Continuous Functions and Homeomorphism, Product Topology

T_0, T_1, T_2 Spaces and Their Characterizations and Basic Properties, Regular and Completely Regular Spaces, Normal and Completely Normal Spaces

Compact / Countable Spaces and Their Properties, Connected Spaces

Texts and Reference Books

1. C. W. Patty, *Foundation of Topology*, 2nd Edition, The Jones and Bartlett Publishers, 2009.
2. J. Dugundji, *Topology*, W.M. C. Brown Publisher, 1990.
3. J. R. Munkres, *Topology: A First Course in Topology*,2nd Edition, Prentice Hall Inc. Englewood Cliffis, New Jessy, 2005.
4. G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata Mc-Graw Hill, 1963.
5. Munkres, *Topology,* 2nd Edition, Pearson India, 2015.

Course Code: MATH-3506 **Title: Abstract Algebra** **Credit Hrs: 03**

Course Outline: Sets and Structures, Basic Axioms and Order of a Group, Subgroups, Subgroups Generated by Subset of a Group, System of Generators, Cyclic Group, Cosets, Lagrange's Theorem, Permutations, Even and Odd Permutations, Cycles, Lengths of Cycles, Transpositions, Symmetric and Alternating Group, Normalizers and Centralizers of a Subset of a Group, Centre of a Group, Normal Subgroup, Quotient Groups, Conjugacy Relation between Elements and Subgroups, Homomorphism and Isomorphism between Groups, Homomorphism and Isomorphism Theorems,

Texts and Reference Books

1. J. B. Fraleigh, *A First Course in Abstract Algebra,* Addison Wesley, 2002.
2. N. Jacobson, *Basic Algebra*, Vol. 1, W.H. Freeman and Company, 1985
3. S. Lang, *Algebra*, Springer-Verlag, 2002.
4. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3rd Edition, Wiley, 2003.
5. J. A. Gallian, *Contemporary abstract algebra*, D.C. Heath, 1986.

Semester-VI

Course Code: MATH-3601 **Title: Real Analysis-II** **Credit Hrs: 03**

Course Outline:

The Riemann-Stieltjes Integrals: Definition and existence of integrals. Properties of integrals. Fundamental theorem of calculus and its applications. Change of variable theorem. Integration by parts. **Functions of Bounded Variation:** Definition and examples. Properties of functions of bounded variation. **Improper Integrals:** Types of improper integrals, tests for convergence of improper integrals. Beta and gamma functions. Absolute and conditional convergence of improper integrals. **Sequences and Series of Functions:** Power series, definition of point-wise and uniform

convergence. Uniform convergence and continuity. Uniform convergence and differentiation. Examples of uniform convergence.

Texts and Reference Books

1. H. L. Royden, *Real Analysis*, Mc-MillanPublishing Company, Inc., New York, 1968.
2. W. Rudin, *Principles of Mathematical Analysis*, Mc-Graw Hill, New York, 1987.
3. T. M. Apostol, *Mathematical Analysis*, 6th Edition, Addison Wesley, 1982.
4. R. G. Bartle and D. R. Sherbert, *Introduction to Real Analysis*, 3rd Edition, John Wiley and Sons, 1999.
5. S. Lang, *Real Analysis*, Springer-Verlag, London, 1987.

Course Code: MATH-3602 Title: Partial Differential Equation Credit Hrs: 03

Course Outline:

First order PDEs: Introduction, formation of PDEs, solutions of PDEs of first order, The Cauchy's problem for quasilinear first order PDEs, First order nonlinear equations, Special types of first order equations, **Second order PDEs:** Basic concepts and definitions, Mathematical problems, Linear operators, Superposition, Mathematical models: The classical equations, the vibrating string, the vibrating membrane, conduction of heat solids, canonical forms and variable, PDEs of second order in two independent variables with constant and variable coefficients, Cauchy's problem for second order PDEs in two independent variables

Methods of separation of variables and Green's Functions: Solutions of elliptic, parabolic and hyperbolic PDEs in Cartesian and cylindrical coordinates

Texts and Reference Books

1. P. Duchaure and D. Zachmann, *Partial Differential Equations*, Mc-Graw Hill, 1986.
2. J. Fritz, *Partial Differential Equations*, Spriger-Verlag, New York, 1979.
3. R. Haberman, *Elementary Applied Partial Differential Equations*, Prentice Hall Inc., 1983.
4. M. Humi and W.B. Miller, *Boundary Value Problems and Partial Differential Equations*, PWS-Kent Publishing Company, Bostan, 1992.
5. E. Zauderer, *Partial Differential Equations of Applied Mathematics*, John Wiley and Sons, Englewood Cliff, New York, 1983.

Course Code: MATH-3603 Title: Numerical Methods-I Credit Hrs: 03

Course Outline: Computer Arithmetic, Approximations and Errors; Methods for the Solution of Non-Linear Equations and Their Convergence: Bisection Method, Regula-Falsi, Fixed Point Iteration, Newton-Raphson, Secant and Householder's Method, Error Analysis for Iterative Methods Interpolation and Polynomial Approximation: Lagrange Interpolation, Newton's Divided Difference, Forward-Difference and Backward-Difference Formulae, Hermite Interpolation, Numerical Integration and Error Estimates: Rectangular Rule, Trapezoidal Rule, Simpson's One-Three and Three-Eight Rules. Numerical Solution of Systems of Algebraic Linear Equations: Gauss-Elimination Method, Gauss-Jordan Method, Matrix Inversion, LU-Factorization, Doolittle's, Crount's, Cholesky's Methods, Gauss-Seidel and Jacobi Methods, Matrix Norms, Method of Least Squares, Eigenvalues and Eigenvectors: Power Method

Texts and Reference Books

1. K. E. Atkinson, *An Introduction to Numerical Analysis*, 2nd Edition, John Wiley and Sons, New York, 1989.
2. R.L. Burden and J.D. Faires, *Numerical Analysis*, 5th Edition, PWS Publishing Company, 1993.
3. S.C. Chapra and R.P. Canale, *Numerical Methods for Engineers*, Mc-Graw Hill, New York, 1988.
4. A. Greenbaum and T. P. Chartier, *Numerical Methods*, 2012 Edition, Princeton University Press, 2012.
5. J. Lambert, *Numerical Methods for Ordinary Differential Systems*, Wiley, 1991.

Course Code: MATH-3604 Title: Complex Analysis Credit Hrs: 03

Course Outline: The Algebra and the Geometry of Complex Numbers, Complex Functions, Parametric Curves in the Complex Plane, Linear Mappings, Special Power Functions, Reciprocal Functions, Limit and Continuity, Complex Functions as Vector Fields, Derivative, Analytic Functions, Cauchy-Riemann Equations, Complex Exponential, Logarithmic, Trigonometric, Hyperbolic Functions and Their Derivatives, Harmonic Functions, Contour Integrals, the Cauchy-Goursat Theorem, Cauchy Integral Formulas, the Morera Theorem, Maximum Modulus Principle, The Liouville Theorem, Fundamental Theorem of Algebra, Convergence of Sequences and Series, The Taylor and the Laurent Series, Uniqueness of Representation, Zeros of Analytic Functions Residues, Poles and the Residue Theorem, Evaluation of Improper Integrals, Integrals Around a Branch Point, The Argument Principle and the Rouche's Theorem

Texts and Reference Books

1. R.V. Churchill and J.W. Brown, *Complex Variables and Applications*, 5th Edition, Mc-Graw Hill, New York, 1989.
2. E. Hille, *Analytic Function Theory*, Vol. I and II, Chelsea Publishing Company, New York, 1974.
3. J. E. Marsden, *Basic Complex Analysis*, W. H. Freeman and Company, 1982.
4. D. G. Zill, and P. D. Shanahan, *A First Course in Complex Analysis with Applications*, Jones and Bartlet Publishers, Sudbury, Massachusetts, 2008.
5. L. V. Ahlfors, *Complex Analysis*, 3rd Edition, McGraw Hill Education (India) Private Limited, 2013.

Course Code: MATH-3605 Title: Tensor Analysis Credit Hrs: 02

Course Outline: Introduction to general tensors, Summation Convention, Kronecker Delta, Levi-Civita Symbol, Vectors as Quantities Transforming Under Rotations with Notation, The Spherical-Polar and the Cylindrical-Coordinate Meshes, Alternating Symbol, Relation between Alternating Symbol and Kronecker Delta, Tensors of First, Second and Higher Orders, Algebra of Tensors, Contraction of Tensor, Quotient Theorem, Symmetric and Skew-Symmetric Tensors, Invariance Property, Isotropic Tensors, Differentiation of Tensors, Application of Tensors in Modeling Anisotropic Systems, Study of Physical Tensors, Diagonalization of Inertia Tensor as Aligning Coordinate Frame with Natural Symmetries of the System

Texts and Reference Books

1. D.E. Bourne and P.C. Kendall, *Vector Analysis and Cartesian Tensors*, 2nd Edition, Thomas Nelson, 1977.
2. G. E. Hay, *Vector and Tensor Analysis*, Dover Publications, Inc., New York, 1979.
3. N.A. Shah, *Vector and Tensor Analysis*, A-One Publishers, Lahore, 2005.

4. I. S. Sokolnikoff, *Tensor Analysis: Theory and Application*, John Wiley and Sons, New York, 1951.
5. J. L. Synge and A. Schild, *Tensor Calculus*, Courier Dover Publications, 1978.
6.

Course Code: MATH-3606 Title: Scientific Programming Credit Hrs: **2+1**

Course Outline (Matlab, Maple, Mathematica)**:** A list of contents is given below. However the instructor may add or remove some of the contents depending on the need of the students.
Curve sketching (only graphs of functions of one variable in Cartesian Coordinates) by using standard procedure, Intro to MATLAB (history, installation, interface, using product help, etc), MATLAB vs Maths syntax, sketching plane curves and sufaces in MATLAB, use of MATLAB in linear algebra, symbolic computations using MATLAB, creating script and function m-files, MATLAB codes for Bisection, Newton-Raphson, secan, Regula-Falsi, Jacobi, GS, Euler, and RK-4 methods. A brief introduction to Maple/Mathematica, comparison between similar features of MATLAB and Maple/Mathematica.
The following points are important for teaching this course:

1. The course should be taught in a computer lab setting
2. At the completion of this course, the students must be able to utilize the software to solve computationally difficult problems
3. The students should have a good command on at least two of the three programs mentioned above

Texts and Reference Books

1. D. M. Etter, D. Kuncicky and D. Hull, *Introduction to Matlab-6*, Prentice Hall, 2001.
2. F. Garvan, *The Maple Book*, Chapman and Hall/CRC, 2002.
3. A. Gilat, *MATLAB*: *An Introduction with Applications*, 5th Edition, Wiley, 2014.
4. L. Nichal, *Maple*, 1st Edition, Nancy Paulsen Books, 2014.
5. S. Kaufmann, *Mathematica as a Tool: An Introduction with Practical Examples*, Springer-Verlag, 1994.
6. Taimoor Salahuddin, Numerical Techniques in MATLAB, Tayler and Francis, 2024.

Semester-VII

Course Code: MATH-4701 Title: Mathematical Physics Credit Hrs: 03

Course Outline: Definition and Properties of Laplace and Inverse Laplace Transforms, s-shifting property of LT, existence and uniqueness theorem, LT of derivatives and integrals and solution of ivps, unite step function and t-shifting property of LT, solution of circuit problems by using LT, Dirac Delta Function and its properties, LT of periodic functions, Convolution Theorem, Convolution of discontinuous functions, solution of integral equations by LT, differentiation and integration of LT, Laguerre's equation and polynomials, Applications of Laplace Transforms to systems of ODEs and PDEs, Fourier Series of 2pi and 2L periodic functions, Convergence and sum of FS, even and odd functions and half-range expansions of FS, solution of odes and pdes by using FS, Fourier Integrals, Fourier Sine and Cosine integrals, Fourier sine and cosine transforms, Fourier sine and cosine transforms of derivatives, existence of FT, FT of derivatives, Convolution theorem, Discrete and Fast Fourier Transforms, solution of odes and pdes by using FT.

Texts and Reference Books

1. E. Kreyszig, *Advanced Engineering Mathematics*, 10th Edition, Wiley, 2011.
2. E. L. Butkov, *Mathematical Physics*, Addison Wesley, 1973.
3. R. P. Kanwal, *Linear Integral Equations*, Academic Press, 1971.
4. T. Myint-U and L. Denbnath, *Partial Differential Equations*, Elsevier Science Publishing, 1987.
5. I. Stakgold, *Boundary Value Problems of Mathematical Physics*, Vol. II, Mc-MillanPublishing Company, Inc., New York, 1968.

Course Code: MATH-4702 **Title:** Functional Analysis **Credit Hrs: 03**

Course Outline: Definition and Examples of Normed Spaces, Banach Spaces, Characterization of Banach Spaces, Bounded Linear Transformations, Bounded Linear Operators, Functional and Their Examples, Various Characterization of Bounded (Continuous) Linear Operator, The Space of All Bounded Linear Operators, The Open Mapping and Closed Graph Theorems, The Dual (Conjugate) Spaces, Reflexive Spaces, Hahn-Banach Theorem (Without Proof), Some Important Consequences of the Hahn- Banach Theorem. Inner Product Spaces, Hilbert Spaces, Orthonormal Bases, Convexity in Hilbert Spaces, Operators in Hilbert Spaces, Invariant Sub-Spaces, Decomposition of Hilbert Spaces, Finite Dimensional Spectral Theory and Spectral Mapping Theorem.

Texts and Reference Books

1. E. Kreyszig, *Introductory Fundamental Analysis with Applications*, John Wiley and Sons, 1978.
2. I. J. Maddox, *Elements of Functional Analysis*, Cambridge University Press, 1970.
3. W. Rudin, *Functional Analysis*, Mc-Graw Hill, 1983.
4. G. F. Simmon, *Introduction to Topology and Modern Analysis*, Mc-Graw Hill, New York, 1983.
5. J. Muscat, *Functional Analysis*, 2014 Edition, Springer, 2014.

Course Code: MATH-4704 **Title: Measure Theory** **Credit Hrs:** 03

Course Outline: Definition and Examples of Algebra and σ-Algebra, Basic Properties of Measurable Spaces, Definition and Examples of Measure Spaces, Outer Measure, Lebesgue Measure, Measurable Sets, Complete Measure Spaces

Measurable Functions: Some Equivalent Formulations of Measurable Functions, Examples of Measurable Functions, Various Characterizations of Measurable Functions, Properties that Hold Almost Everywhere

Definition of Lebesgue Integral, Basic Properties of Lebesgue Integrals, Comparison between Riemann Integration and Lebesgue Integration, L_2-Spaces

Texts and Reference Books

1. D. L. Cohn, *Measure Theory*, Birkhauser, 1980.
2. P. R. Halmos, *Measure Theory*, D. Van Nostrand, 1950.
3. H. L. Royden, *Real Analysis*, Mc-MillanPublishing Company, Inc., New York, 1968.
4. T. Tao, *An Introduction to Measure Theory*, American Mathematical Society, 2011.
5. P. R. Halmos, *Measure Theory*, 2nd Edition, Springer, 1978.

Course Code: MATH-4705 Title: Numerical Methods-II Credit Hrs: 03

Course Outline: IVP for ODEs: Elementary theory of ivps, Introduction to Single-Step Numerical Methods for solution of ivps, Picard's method, Euler's Method, Second and higher order Taylor's methods, RK-methods (mid-point, modified Euler, Heun's, and RK-4 methods), Multi-Step methods for numerical solutions of odes (2-, 3-, 4-, and 5-step Adam-Bashforth and Adam-Moulton methods), Extension of Euler and RK-4 methods for numerical solutions of higher order odes and system of first order odes, stability, consistency, and convergence of SS and MS methods, stiff odes, region of absolute stability and A-Stability,

BVPs for odes: Discussion on Problem Behavior and Stability, Shooting method and finite difference methods for Linear and Non-Linear BVPs, Rayleigh-Ritz method for Linear and Non-Linear BVPs.

Texts and Reference Books

1 U. Ascher and L. Petzold, *Computer Methods for ODEs and DAEs*, SIAM, 1998.

2 R.L. Burden and J.D. Faires, *Numerical Analysis*, 5th Edition, PWS Publishing Company, 1993.

3 U. Ascher, R. Matteij and R. Russell, *Numerical Solution of BVPs for ODEs*, Prentice Hall, 1988.

4 J. Lambert, *Numerical Methods for Ordinary Differential Systems*, Wiley, 1991.

5 A. Greenbaum and T. P. Chartier, *Numerical Methods*, 2012 Edition, Princeton University Press, 2012.

Course Code: MATH-4706 Title: Fluid Mechanics-I Credit Hrs: 03

Course Outline: Fields and Continuum Concepts, Lagrangian and Eulerian Specifications, Local, Convective and Total Rates of Change, Conservatoin of Mass, Equation of Continuity, Boundary Conditions, Nature of Force in a Fluid Field and Their Effects: Surface and Body Forces, Stress at a Point, Viscosity and Newton's Viscosity Law, Viscous and Inviscid Flows, Laminar and Turbulent Flows, Compressible and Incompressible Flows

Irrotational Fluid Motion: Velocity Potential from an Irrotational Velocity Field, Streamlines. Vortex Lines and Vortex Sheets, Kelvins Minimum Energy Theorem, Conservation of Linear Momentum, Bernoullis Theorem and Its Applications, Circulations, Rate of Change of Circulation (Kelvins Theorem), Axially Symmetric Motion, Stokes Stream Function

Two-Dimensional Motion: Stream Function, Complex Potential and Complex Velocity, Uniform Flows, Sources, Sinks and Vortex Flows, Flow in a Sector, Flow Around a Sharp Edge, Flow Due to a Doublet

Texts and Reference Books

1. I. G. Curie, *Fundamentals of Mechanics of Fluids*, 3rd Edition, CRC, 2002.
2. I. L. Distsworth, *Fluid Mechanics*, Mc-Graw Hill, 1972.
3. R.W. Fox, A.T. Mc-Donald and P.J. Pritchard, *Introduction to Fluid Mechanics*, John Wiley and Sons, 2003.
4. H. Schlichting, K. Gersten, E. Krause and H. Oertel, Jr, *Boundary-Layer Theory*, 8th Edition, Springer-Verlag, 2004.
5. Y. C-Shun, *Fluid Mechanics*, Mc-Graw Hill, 1974.

Course Code: MATH-4707 Title: Discrete Structures Credits: 3

Course Outlines: Set and Relations:Basic Notions, Set Operations, Venn Diagrams, Extended-Set Operations, Indexed Family of Sets, Countable and Uncountable Sets, Relations, Cardinality, Equivalence Relations, Congruence, Partitions, Partial Order, Representation of Relations, Mathematical Induction

Elementary Logic:Logics of Order Zero and One, Propositions and Connectives, Truth Tables, Conditionals and Bi-Conditionals, Quantifiers, Methods of Proof, Proofs Involving Quantifiers

Texts and Reference Books

1. K. H. Rosen, *Discrete Mathematics and its Applications*, 12th Edition, Mc-Graw Hill, New York, 1999.
2. K. A. Ross and C. R. B. Wright, *Discrete Mathematics*, Prentice Hall, Englewood Cliffs, NJ, USA, 2003.
3. H. Fell and J. A. Aslam, *Discrete Structures*, 5th Edition, Cognella Academic Publishing, 2016.
4. S. S. Epp, *Discrete Mathematics with Applications*, 4th Edition, Brooks Cole, 2010.
5. B. Kolman and R. C. Busby, *Discrete Mathematical Structures*, 6th Edition, Pearson, 2017.

Course Code: MATH-4708 Title: Special Functions Credit Hrs: 03

Course Outline: Infinite Products: introduction, Definition of an infinite product, A necessary condition for convergence, Associated series of logarithms, Convergences types, The Gamma and Beta Functions: The Euler constant, The Gamma function, A series for logarithmic differential of Gamma function, The order symbols o and O, Evaluation of certain infinite products,The Beta Function, Factorial function, Asymptotic Series: Definition of an asymptotic expansion, Algebraic properties, Term by term integration, Uniquenesss, The Hypergeometric function : Simple integral form, The function F(a,b;c,z) and its properties, The Hypergeometric differential equation, Logarithmic solution of the hypergeometric function, F(a,b;c,z) as a function of parameters, Elementary series manipulation, Kummer Theorem, Generalized Hypergeometric functions, the exponential and binomial functions, A differential equation,Saalschutz theorem, Contour integral integrals of Barnes' type, the Barnes integrals and the generalized hypergeometric function, Bessel Functions and its properties, The Confluent Hypergeometric functions and its properties, Generating Functions.

Texts and Reference Books

1. E. D. Rainville, *Special Funcgions*, 1st Edition, The Macemillean Company New York, 1965.
2. L. J. Slater, and D. Lit, *Confluent Hyper geometric Functions*, Cambridge University Press, London, 1960.
3. A. Erdeyli, *Higher Transcendental Functions*, Volume I, II, & III, McGraw Hill Book Company, Inc. 1953.
4. J. E. Andrews, R. Asky and R. Roy, *Special Functions*, 1st Edition, Cambridge University Press, 2001.
5. C. Viola, *Special Functions*, 1st Edition, Springer, 2016.

Course Code: MATH-4709 Title: Quantum Mechanics-I Credit Hrs: 03

Course Outline: Black Body Radiation, Photoelectric Effect, Compton Effect, Bohr's Theory of Atomic Structure, Wave-Particle Duality, The De Broglie Postulate, Heisenberg Uncertainty

Principle, The Postulates of Quantum Mechanics, Operators, Eigen-functions and Eigenvalues: Observables and Operators. Measurement in Quantum Mechanics, The State Function and Expectation Values Time, Development of the Stat Function (Schrodinger Wave Equation), Solution to the Initial-Value Problem in Quantum Mechanics, Parity Operators

Function Spaces and Hermitian Operators: Particle in a Box, Dirac Notation, Hilbert Space. Hermitian Operators, Properties of Hermitian Operators, Additional One-Dimensional Problems: Bound and Unbound States: General Properties of the One Dimentional Schrodinger Equation, Unbound States, One-Dimensional Barrier Problems, The Rectangular Barrier, Tunneling

Texts and Reference Books

1. D. R. Bes, *Quantum Mechanics: A Modern a Concise Introductory Course*, Springer-Verlag, 2004.
2. H. D. Dehmen, *The Picture Book of Quantum Mechanics*, Springer-Verlag, 2001.
3. H. F. Hameks, *Quantum Mechanics: A Conceptual Approach*, Wiley- IEEE, 2004.
4. R. L. Liboff, *Introduction Quantum Mechanics*, Addison Wesley Publishing Co., 2003.
5. V. K. Thankappan, *Quantum Mechanics*, New Age Publishers, 1993.

Course Code: MATH-4710 Title: Ring Theory Credit Hrs: 03

Course Outline:

Rings: Definition, examples. Quadratic integer rings. Examples of non-commutative rings. The Hamilton quaternions. Polynomial rings. Matrix rings. Units, zero-divisors, nilpotents, idempotents. Subrings, Ideals. Maximal and prime Ideals. Left, right and two-sided ideals;.

Operations with ideals. The ideal generated by a set. Quotient rings. Ring homomorphism. The isomorphism theorems, applications. Finitely generated ideals. Rings of fractions.

Integral Domain: The Chinese remainder theorem. Divisibility in integral domains, greatest common divisor, least common multiple. Euclidean domains. The Euclidean algorithm. Principal ideal domains. Prime and irreducible elements in an integral domain. Gauss lemma, irreducibility criteria for polynomials. Unique factorization domains. Finite fields. Polynomials in several variables. Symmetric polynomials. The fundamental theorem of symmetric polynomials.

Texts and Reference Books

1. J. B. Fraieigh, *A First Course in Abstract Algebra*, Addison Wesley, 2002.
2. B. Hartley and T. O. Hawkes, *Ring, Modules and Linear Algebra*, Chapman and Hall, 1980.
3. S. Lang, *Algebra*, Springer-Verlag, 2002.
4. S. J. Leon, *Linear Algebra with Applications*, 6th Edition, Prentice Hall, 2002.
5. P. M. Cohn, Introduction to *Ring Theory*, 1st Edition, Springer, 2002.

Course Code: MATH-4711 Title: Analytical Dynamics Credit Hrs: 03

Course Outline: Constraints, Generalized Coordinates, Generalized Forces, General Equation of Dynamics, Lagrange's Equations, Conservation Laws, Ignorable Coordinates, Explicit Form of Lagrange's Equation in Terms of Tensors. Hamilton' Principle of Least Action, Hamilton's Equations of Motion, Hamilton-Jacobi Method, Poisson Brackets (P.B's); Poisson's Theorem; Solution of Mechanical Problems by Algebraic Technique Based on (P.B's) Small Oscillations and Normal Modes, Vibrations of Strings, Transverse Vibrations Normal Modes, Forced Vibrations and Damping, Reflection and Transmission at a Discontinuity, Longitudinal Vibrations, Rayleigh's Principle

Texts and Reference Books

1. F. Chorlton, *Textbook of Dynamics*, Van Nostrand, 1963.
2. W. Chester, *Mechanics*, George Allen and Unwin Ltd. London, 1979.
3. H. Goldstein, *Classical Mechanics*, Cambridge University Press, 1980.
4. G. Meirovitch, *Methods of Analytical Dynamics*, Mc-Graw Hill, 1970.
5. H. Baruh, *Analytical Dynamics*, 1st Edition, WCB/McGraw-Hill, 1998.

Course Code: MATH-4712 Title: Approximation Theory Credit Hrs: 03

Course Outline: Best Approximations in Normal Spaces, Existence Theorem, Approximation by Algebraic Polynomials, Uniqueness, Strict Convexity, Lemma Convexity, Uniqueness Theorem, Hilbert Space, Uniform Approximation, External Point, Haar Condition, Extreme Points, Best Approximation, Characterization of Best Approximation, A Brief Introduction to Interpolation, Chebyshev Polynomials, Chebyshev Theorem, Least Squares Approximation, Gram-Schmidt Orthonormalization Process, Orthogonal Function, Pade Approximation, Remex Algorithm

Texts and Reference Books

1. N. I. Achieser, *Theory of approximation*, Dover Publications, Inc., New York, 2004.
2. J. R. Rice, *The Approximation of Functions*, Vol. I, Addison Wesley Publishing Company, 1964.
3. T. J. Rivlin, *An Introduction to the Approximation of Functions*, Dover Publications, Inc., New York, 1981.
4. M. J. D. Powell, *Approximation Theory and Methods*, Cambridge University Press, 1988.
5. E. W. Cheney, *Introduction to Approximation Theory*, AMS, 2000.

Course Code: MATH-4713 Title: Differential Geometry-II Credit Hrs: 03

Course Outline: Definition and Examples of Manifolds, Differential Maps, Sub-Manifolds, Tangents, Coordinate Vector Fields, Tangent Spaces, Dual Spaces, Multi-Linear Functions, Algebra of Tensors, Vector Fields, Tensor Fields, Integral Curves, Flows, Lie Derivatives, Brackets, Differential Forms, Introduction to Integration Theory on Manifolds, Riemannian and Semi-Riemannian Metrics, Flat Spaces, Affine Connections, Parallel Translations, Covariant

Differentiation of Tensor Fields, Curvature and Torsion Tensors, Connection of a Semi-Riemannian Tensor, Killing Equations and Killing Vector Fields, Geodesics, Sectional Curvature

Texts and Reference Books

1. R. Abraham, J. E. Marsden and T. Ratiu, *Manifolds, Tensor Analysis and Applications*, Addison Wesley, 1983.
2. R. L. Bishop and S. I. Goldberg, *Tensor Analysis on Manifolds*, Dover Publications, Inc., New York, 1980.
3. M. P. Do Carmo, *Riemannian Geometry*, Birkhauser, 1992.
4. D. Langwitz, *Differential and Riemannian Geometry*, Academic Press, 1970.
5. E. Kreysizg, *Differential Geometry*, 1st Edition, Dover Publications, 1991.

Course Code: MATH-4714 Title: Electromagnetic Theory-I Credit Hrs: 03

Course Outline: Electromagnetic Fields, Coulomb's Law, The Electric Field Intensity and Potential, Gauss's Law and Deductions, Piosson and Laplace Equations, Conductors and Condensers, Dipoles, The Linear Quadrupole, Potential Energy of a Charge Distribution, Dielectrics, The Polarization and Displacement Vectors, General Solutions of Laplace's Equation. Solutions of Laplace's Equation in Spherical Coordinates, Legendre's Equation, Legendre's Polynomials, Electromagnetic Fields, The Magnetostatic Law of Force, The Magnetic Induction, The Lorentz Force on a Point Charge Moving in a Magnetic Field, The Divergence of the Magnetic Field, The Vector Potential, The Conservation of Charge and Equation of Continuity, The Lorentz Condition, The Curl of The Magnetic Field, Ampere's Law and the scalar Potential, Steady and Slowly Varying Currents, Electric Current, Linear Conductors. Conductivity, Resistance, Kirchhoff's Laws, Current Density Vector, Magnetic Field of Straight and Circular Current, Magnetic Flux, Vector Potential, Forces on a Circuit in Magnetic Field

Texts and Reference Books

1. D. Corrisonand P. Lorrison, *Introduction to Electromagnetic Fields and Waves* , W.H. Freeman and company, London, 1962.
2. D. J. Griffiths, *Introduction to Electrodynamics*, Prentice-Hall, 1999.
3. J. D. Jackson, *Classical Electrodynamics*, Wiley, 1999
4. G. E. Owen, *Introduction to ElectromagneticTheory*, Dover, 2003.
5. J. R. Reitz, F. J. Milford and R. W. Christy, *Foundations, of Electromagnetic Theory*, Addison-Wesley Publishing, 1993.

Course Code: MAT-4715 Title: Introduction to Univalent Functions Credit Hrs: 3

Course Outline: 1. Basic Concepts: **Complex Analysis Review:**A brief refresher on essential concepts from complex analysis, such as analytic functions, Cauchy-Riemann equations, and conformal mappings. **Univalent Functions Definition:** Formal definition of univalent (or injective) functions and their significance in complex analysis. Exploring Möbius transformations and their role as automorphisms of the unit disk. A crucial theorem stating that any simply connected domain (other than the whole plane) can be conformally mapped to the unit disk. Properties of Univalent Functions: **Area Theorem:** A fundamental theorem providing an upper bound on the area of the image of a univalent function. Studying bounds on the Taylor series coefficients of univalent functions. Analyzing how univalent functions affect the size and shape of geometric figures. **Extremal Problems:**
Investigating problems where one seeks to find the maximum or minimum value of a functional defined on a class of univalent functions. **Boundary Behavior:** Examining the behavior of univalent functions near the boundary of their domain, including angular

limits and derivatives. Special Classes of Univalent Functions: Functions whose images are starlike with respect to a point. Functions whose images are convex sets. A broader class of univalent functions related to convex functions. Functions that map the upper half-plane to itself. Advanced Topics (Time Permitting): A powerful technique for constructing and analyzing univalent functions, particularly those with slit images. A famous conjecture (now a theorem) about the coefficient estimates for univalent functions. Studying the behavior of univalent functions using integral means.

Texts and Reference Books

1. Derek K. Thomas (Author), Nikola Tuneski, Univalent Functions: A Primer (De Gruyter Studies in Mathematics.
2. James A. Jenkins, Univalent Functions and Conformal Mapping.

Semester-VIII

Course Code: MATH-4801 Title: Integral Equations Credit Hrs: 03

Course Outline: Linear integral equations of the first and second kind. Relationship between differential equation and Volterra integral equation. Neumann series. Fredholm Integral equation of the second kind with separable Kernels. Eigenvalues and eigenvectors. Iterated functions. Quadrature methods. Least square methods. Homogeneous integral equations of the second kind. Fredholm integral equations of the first kind. Fredholm integral equations of the second kind. Abel's integral equations. Hilbert Schmidt theory of integral equations with symmetric Kernels. Regularization and filtering techniques, Introduction to Wiener-Hopf Technique.

Texts and Reference Books

1. J. J. Abdul, *Introduction to Integral Equations with Applications*, Marcel Dekker Inc. New York, 1985.
2. V. Lovitt, *Linear Integral Equations*, Dover Publications, Inc., New York, 1950.
3. B. Noble, *Methods Based on the Wiener-Hopf Technique*, Pergamon Press, 1988
4. F. Smith, *Integral Equations*, Cambridge University Press, 2003
5. F. G. Tricomi, *Integral Equations*, Interscience, 1957.

Course Code: MATH-4803 Title: Optimization Theory Credit Hrs: 03

Course Outline: Linear Programming: Simplex Method, Duality Theory, Dual and Primal-Dual Simplex Methods Unconstrained Optimization: Optimality Conditions, One-Dimensional Problems, Multi-Dimensional Problems and the Method of Steepest Descent, Constrained Optimization with Equality Constraints, Optimality Conditions, Lagrange Multipliers, Hessians and Bordered Hessians, Inequality Constraints, the Kuhn-Tucker Theorem and Applications

Texts and Reference Books

1. L. Elsgolts, *DEs and the Calculus of Variations*, Mir Publishers Moscow, 1970.
2. B. S. Gotfried and J. Weisman, *Introduction to Optimization Theory*, Prentice Hall, Englewood Cliffs, NJ, USA, 1973.

3. D. G. Luenberger, *Introduction to Linear and Non-Linear Programming*, Addision Wesley, Reading, Ma. USA, 1973.
4. H. T. Jongen and K. Meer, *Optimization Theory*, 2004 Edition, Springer, 2004.
5. A. Beck, *Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with MATLAB*, SIAM-Society for Industrial and Applied Mathematics, 2014.

Course Code: MATH-4804 Title: Fluid Mechanics-II Credit Hrs: 03

Course Outline: Circular Cylinder without Circulation, Circular Cylinder with Circulation Blasius Theorem, Kutta Condition and the Flat-Plate Airfoil, Joukowski Airfoil, Vortex Motion, Karman's Vortex Street, Method of Images, Velocity Potential, Stoke's Stream Function, Solution of the Potential Equation, Uniform Flow Source and Sink, Flow Due to A Doublet Viscous Flows of Incompressible Fluids: Constitutive Equations, Navier-Stokes's Equations, Exact Solutions of Navier-Stokes's Equations, Steady Unidirectional Flow, Poiseuille Flow, Coquette Flow, Flow between Rotating Cylinders, Stoke's First Problem, Stoke's Second Problem
Simplified Approach to Fluid Flow Problems: Similarity from Differential Equations, Dimensional Analysis, One Dimensional Steady Compressible Flow

Texts and Reference Books

1. I. G. Curie, *Fundamentals of Mechanics of Fluids*, 3rd Edition, CRC, 2002.
2. I. L. Distsworth, *Fluid Mechanics*, Mc-Graw Hill, 1972.
3. R.W. Fox, A.T. Mc-Donald and P.J. Pritchard, *Introduction to Fluid Mechanics*, John Wiley and Sons, 2003.
4. H. Schlichting, K. Gersten, E. Krause and H. Oertel, Jr, *Boundary-Layer Theory*, 8th Edition, Springer-Verlag, 2004.
5. Y. C-Shun, *Fluid Mechanics*, Mc-Graw Hill, 1974.

Course Code: MATH-4802 Title: Capstone Project Credit Hrs: 03

Course Outline:
The objective of this course is to train students to learn and use the tools required for writing their project report and the tools for doing their research work. They will do the literature survey, work on some research problem and write a project report by the end of the course.

Texts and Reference Books

As per requirements of the topic of the Project.

Course Code: MATH-4805 Title: Mathematical Modeling Credit Hrs: 03

Course Outline: Introduction to modeling. Basic Guide line of mathematical modeling, Collection and interpretation of data, Technique of mathematical modeling, Classification of mathematical modeling, modeling through algebra, modeling through Geometry, modeling through Trigonometry, modeling through Calculus, Limitation of modeling, Development of Models', **,** Discrete and Continuous models,, Linear Growth and Decay Model, Non-linear Growth and decay models, Mathematical modeling in population dynamic, Traffic flow models.

Texts and Reference Books

1. D. Edwards and M. Hamson, *Mathematical modeling Skills*, MacMillan Press Ltd., 1996
2. F. R. Giordano, M. D. Weir, and W. P. Fox, *A First Course in Mathematical modeling*, Thomson Brooks/Cole, 2003
3. J. N. Kapur, *Mathematical modeling*, New Age International Limited, 2015.
4. S. Banerjee, *Mathematical modeling* (Modell analysis and Application)
5. J. J. Batzel, M. Bachar, and F. Kappel, *Mathematical Modeling and Validation in Physiology*, 2013

Course Code: MATH-4806 Title: Introduction to Fractional Calculus Credit Hrs: 03

Course Outline: Foundations and Preliminaries: Introduction to Fractional Calculus: Review of Integer-Order Calculus: Brief review of differentiation and integration with integer orders, setting the stage for generalization. Special Functions: Introduction to Gamma, Beta, and Mittag-Leffler functions, which are essential for fractional calculus. Laplace and Fourier Transforms: ractional Derivatives and Integrals: Riemann-Liouville Fractional Integral: Definition and properties. Riemann-Liouville Fractional Derivative: Definition and properties, including the relationship between Riemann-Liouville integrals and derivatives. Definition and properties, with a focus on its advantages in applications. Grunwald-Letnikov Fractional Derivative: An alternative definition and its relation to other definitions. Definition and properties of Weyl fractional integrals and derivatives.

Texts and Reference Books

1. Xiao-Jun Yang, General Fractional Derivatives: Theory, Methods and Applications.
2. Samko, Kilbas,Fractional Integrals and Derivatives: Theory and Applications.

Course Code: MAT-4807 Title: Algebraic Topology Credit Hrs: 03

Course Outline: Path Wise Connectedness with Examples, Notion of Homotopy, Homotopy Classes and its Application, Path Homotopy, Path Homotopy Classes, Fundamental Groups, Covering Mapping with Examples, Covering Spaces, Lifiting Properties of Covering Spaces and its Application, Fundamental Group of a Circle (s)

Text and Reference Books

1. M. J. Greeberge, *Algebraic Topology*, A first Course, Benjamin Commings, 1967.
2. C.A. Kosniowski, *First course in Algebraic Topology*, Cambridge University press, 1980.
3. A. H. Wallace, *Algebraic Topology*, Homology and Cohomology, Benjamin, 1968.
4. A. Hatcher, *Algebraic Topology*, 1st Edition, Cambridge University Press, 2001.
5. W. Fulton, *Algebraic Topology*, Springer, 1997.

Course Code: MATH-4808 Title: Mathematical Systems Theory Credit Hrs: 03

Course Outline: Conservation laws and phenomenological principles, some principles and laws of thermodynamics, mechanics, and electromagnetism and their applications in modelling, Linearization, matrix exponentials and solution of linear differential equations, LTV systems, Impulse and step responses, Stability, controllability, and observability of LTI systems, Realization theory and Hankel matrices, Feedback and stabilizability, observers and state reconstruction, detectability, separation principle and compensation, disturbance rejection,
Laplace transforms and LTI systems, transfer functions and transfer matrices, transfer functions and minimal realizations of SISO and MIMO systems, Abstract systems description and behavioral modeling, polynomial representations of systems, brief discussion of nonlinear, descriptor, stochastic, distributed parameter, and discrete event systems, optimal control theory, parameter estimation, filter theory, model reduction, and adaptive and robust control

Texts and Reference Books

1. G. J. Olsder, J. W. van der Woude, J. G. Mask, and D. Jeltsema, *Mathematical Systems Theory*, 4th Edition, VSSD, 2011.
2. I. D. Hinrichsen and A. J. Pritchard, *Mathematical Systems Theory*, Springer
3. J. W. Polderman and J. C. Willems, *Introduction to Mathematical Systems Theory*, 2nd Edition, Springer, 2008.
4. J. Zabczyk, *Mathematical Control Theory*, 1st Edition, Birkhauser, 2007.
5. E. D. Sontag, *Mathematical Control Theory*, 2nd Edition, Springer, 1998.

Course Code: MAT-4809 Title: Dynamical Systems Credit Hrs: 03

Course Outline: Introduction: Preliminary ideas, Autonomous equations, Autonomous systems in plane, Flows and evolution. Linear systems: Linear changes of variables, Similarity types for 2×2 real matrices, Phase portraits for canonical systems in the plane, Classification of simple linear phase portraits in the plane, The evolution operators, Affine systems, Linear systems of dimension greater than two. Nonlinear systems in the plane: Local and global behavior, Linearization at a fixed point, The linearization theorem, Non-simple fixed points, Stability of fixed points, Ordinary points and global behavior, First integrals, Limit points and limit cycles. Flows on non-planar phase spaces: Fixed points, Closed orbits, Attracting sets and attractors, Further integrals. Applications: Linear models, Affine models, Nonlinear models, Relaxation oscillation, Piecewise modeling. Dynamical systems with mathematica: Differential equations, Planar systems, Interacting species, Limit cycles.

Text and Reference Books

1. Lynch S., *Dynamical systems with applications using mathematica*, Birkhauser Boston 2007.
2. Alligood T. K., Sauer D. T., Yorke A. J., *Chaos: An introduction to dynamical systems*, Springer 1996.
3. R. C. Robinson, *An introduction to dynamical systems*, 1st edition, Prentice Hall, 2004.
4. Arrowsmith K. D., Place M. C., *Dynamical systems, differential equations, maps and chaotic behavior*, Chapman & Hall, 1992.
5. D. K. Arrowsmith and C. M. Place, *An introduction to dynamical systems*, Cambridge University Press, 1st edition, 1990.

Course Code: MATH-4810 Title: Quantum Mechanics-II Credit Hrs: 03

Course Outline: Harmonic Oscillator and Problems in Three-Dimensions: The Harmonic Oscillator, Eigen-Functions of the Harmonic Oscillator, The Harmonic Oscillator in Momentum Space, Motion in Three Dimensions, Spherical Symmetric Potential and the Hydrogen Atom, Angular Momentum: Basic Properties, Eigenvalues of the Angular Momentum Operators, Eigen-Functions of the Orbital Angular Momentum Operators L_2 and Lz, Commutation Relations between Components of Angular Momentum and Their Representation in Spherical Polar Coordinates

The Scattering Cross-Section, Scattering Amplitude, Scattering Equation, Born Approximation, Partial Wave Analysis, Time Independent Perturbation of Non-Degenerate and Degenerate Cases, Time-Dependent Perturbations

Texts and Reference Books

1. D. R. Bes, *Quantum Mechanics: A Modern a Concise Introductory Course*, Springer-Verlag, 2004.
2. H. D. Dehmen, *The Picture Book of Quantum Mechanics*, Springer-Verlag, 2001.
3. H. F. Hameks, *Quantum Mechanics, A Conceptual Approach*, Wiley- IEEE, 2004.
4. R. L. Liboff, *Introduction Quantum Mechanics*, Addison Wesley Publishing Company, 2003.
5. V. K. Thankappan, *Quantum Mechanics*, New Age Publishers, 1993.

Course Code: MATH-4811 Title: Introduction to Combinatorics Credit Hrs: 03

Course Outline: Two Basic Counting Principle, Permutations, Combinations, The Injective and Bijective Principles, Arrangements and Selections with Repetitions, Graphs in Combinatorics, The Binomial Theorem, Combinatorial Identities, Properties of Binomial Coefficients, Multinomial Coefficients, The Multinomial Theorem, The Pigeonhole Principle, Examples, Ramsay Numbers, The Principle of Inclusion and Exclusion, Generalization, Integer Solutions, Surjective Mapping. Stirling Numbers of the Second Kind, The Sieve - Eratosthenes, Euler Φ–Function, The Problem Des-Menages

Ordinary Generating Functions, Modeling Problems, Partition of Integers, Exponential Generating Functions

Linear Homogeneous Recurrence Relations, Algebraic Solutions of Linear Recurrence Relations and Constant Functions, The Method of Generating Functions, A Non-Linear Recurrence Relation and Catalaa Numbers

Texts and Reference Books

1. V. K. Balakrishnan, *Theory and Problems of Combinatorics*, Schaum's Outline Series, Mc-Graw Hill, Singapore, 1995.
2. C. C. Chen and K. M. Koh, *Principles and Techniques in Combinatorics*, World Scientific Publishing Company, Pvt. Ltd, Singapore, 1992.
3. C. L. Liu, *Introduction to Combinatorial Mathematics*, Mc-Graw Hill, New York, 1968.
4. A. Tucker, *Applied Combinatorics*, John Wiley and Sons, New York, 1985.
5. R. A. Brualdi, *Introductory Combinatorics*, 5th Edition, Pearson, 2009.

Course Code: MATH-4812 Title: Variational Inequalities Credit Hrs: 03

Course Outline: Variational Inequalities in Fixed Points, The Characterization of the Projection onto a Convex Set, A First Theorem about Variational Inequalities, Variational Inequalities, Some Problems which Lead to Variational Inequalities, Variational Inequalities in Hilbert Space, The Obstacle Problem, Variational Inequalities for Monotone Operators, Penalization

Texts and Reference Books

1. C. Baiocchi and A. Capelo, *Variational and Quasi-Variational Inequalities*: *Applications to Free Boundary Problems*, John Wiley and Sons, New York, 1984.
2. V. Barbu, *Optimal Control of Variational Inequalities*, Pitman Adv. Pub. Program, Boston, 1984.
3. G. Isac, *Complementarity Problems and Variational Inequalities*, Springer-Verlag, 2006.
4. D. Kinderlehrer and G. Stampacchia, *An Introduction to Variational Inequalities and Their Applications*, Academic Press, New York, 1980.
5. M. A. Noor, *Principles of Variational Inequalities*, Lambert Academic Publishing, Saarbrucken, Germany, 2009.

Course Code: MATH-4813 Title: Theory of Elasticity Credit Hrs: 03

Course Outline: Cartesian Tensors, Analysis of Stress and Strain, Generalized Hooke's Law, Crystalline Structure, Point Groups of Crystals, Reduction in the Number of Elastic Moduli Due to Crystal Symmetry, Equations of Equilibrium, Boundary Conditions, Compatibility Equation, Plane Stress and Plane Strain Problems, Two Dimensional Problem in Rectangular and Polar Coordinates, Torsion of Rods and Beams

Texts and Reference Books

1. E. Dieulesaint and D. Royer, *Elastic Waves in Solids*, John Wiley and Sons, New York, 1980.
2. Y. C. Funk, *Foundations of Solid Mechanics*, Prentice-Hall, Englewood Cliffs, 1965.
3. Sokolinikoff, *Mathematical Theory of Elasticity*, Mc-Graw Hill, New York.
4. S. P. Temoshenko and J. N. Goodier, *Theory of Elasticity*, 3rd Edition, Mcgraw Hill Education (India) Private Limited, 2015.
5. A. I. Lurie and A. Belyaev, *Theory of Elasticity*, 2005 Edition, Springer, 2005.

Course Code: MATH-4814 Title: Electromagnetic Theory-II Credit Hrs: 03

Course Outline: The Faraday Induction Law, Induced Electromotance in a Moving System, Inductance and Induced Electromotance, EnerguStoted in a Magnetic Field, The Equations of Electromagnetism, Electromagnetic Waves: Plane Electromagnetic Waves in Free Space and Isotropic Media, The Poynting Vector in Free Space, Propagation of Electromagnetic Waves in NON-Conductors, Propagation of Plane Waves in Conducting Media, Reflection and Reffraction of Plane Waves, Guided Waves, Coaxial Line, Hollow Rectangular Wave Guide, Radiation of Electromagnetic Waves, Electromagnetic Field of a Moving Charge

Texts and Reference Books

1. D. Corrison and P. Lorrison, *Introduction to Electromagnetic Fields and Waves* , W.H. Freeman and company, London, 1962.

2. D. J. Griffiths, *Introduction to Electrodynamics*, Prentice-Hall, 1999.
3. J. D. Jackson, *Classical Electrodynamics*, Wiley, 1999
4. G. E. Owen, *Introduction to ElectromagneticTheory*, Dover, 2003.
5. J. V. Stewart, *Intermediate Electromagnetic Theory*, World Scientific, 2001.

Course Code: MATH-4815 Title: Special Theory of Relativity Credit Hrs: 03

Course Outline: Historical Background and Fundamental Concepts of Special Theory of Relativity, Lorentz Transformations (for Motion Along One Axis), Length Contraction, Time Dilation and Simultaneity, Velocity Addition Formulae, 3-Dimensional Lorentz Transformation, Introduction to 4-Vector Formalism, Lorentz Transformations in the 4-Vector Formalism, The Lorentz and Poincare Groups, Introduction to Classical Mechanics, Minkowski Space-Time and Null Cone, 4-Velocity, 4-Momentum and 4-Force, Application of Special Relativity to Doppler Shift and Compton Effect, Particle Scattering, Binding Energy, Particle Production and Decay, Electromagnetism in Relativity, Electric Current, Maxwell's Equations and Electromagnetic Waves, The 4-Vector Formulation of Maxwell's Equations, Special Relativity with Small Acceleration

Texts and Reference Books

1. H. Goldstein, *Classical Mechanics*, Addison Wesley, New York, 1962.
2. D. Inverno, *Introducing Einstein's Relativity*, Oxford University Press, 1992.
3. J. D. Jackson, *Classical Electrodynamics*, John Wiley and Sons, New York, 1962.
4. A. Qadir, *Relativity: An Introduction to the Special Theory*, World Scientific, 1989.
5. W. Rindler, *Essential Relativity*, Springer-Verlag, 1977.

1(b) Approval of Degree Title and Scheme of studies for BS (Lateral Entry)

The term lateral entry will be used with the session e.g. Session 2022-2024 (Lateral Entry)

Nomenclature (Degree Title): Bachelor of Science in Mathematics (Lateral Entry)
Semesters: **4-8**
Total Credit Hours: **65**
Session: **2023 and onwards**

Scheme of Studies, Bachelor of Science in Mathematics, Sessions 2022-2024 (Lateral Entry) and onwards

Bridging Semester/Courses (For BSc/ADS Annual System)

Four to six bridging courses (12-18 Credit Hours) will be offered as deficiency courses. The deficiency courses will be decided by the Department and may be offered in the bridging semester or with regular semesters as additional courses.

Semester-V

MATH-3501	Real Analysis-I	3	0	3
MATH-3502	Differential Equations II	3	0	3
MATH-3503	Differential Geometry-I	3	0	3
MATH-3504	Analytical Mechanics	3	0	3
MATH-3505	Topology	3	0	3
MATH-3506	Abstract Algebra	3	0	3

Semester-VI

MATH-3601	Real Analysis-II	3	0	3
MATH-3602	Partial Differential Equations	3	0	3
MATH-3603	Numerical Methods-I	3	0	3
MATH-3604	Complex Analysis	3	0	3
MATH-3605	Tensor Analysis	2	0	2
MATH-3606	Scientific Programming	2	1	3

Semester-VII

MATH-4701	Mathematical Physics	3	0	3
MATH-4702	Functional Analysis	3	0	3
MATH-4703	Internship /Field Experience	3	0	3

Two Optional Courses

MATH-4704	Measure Theory	3	0	3
MATH-4705	Numerical Methods II	3	0	3
MATH-4706	Fluid Mechanics-I	3	0	3
MATH-4707	Discrete Structures	3	0	3
MATH-4708	Special Functions	3	0	3
MATH-4709	Quantum Mechanics-I	3	0	3
MATH-4710	Ring Theory	3	0	3
MATH-4711	Analytical Dynamics	3	0	3
MATH-4712	Approximation Theory	3	0	3
MATH-4713	Differential Geometry-II	3	0	3
MATH-4714	Electromagnetic Theory-I	3	0	3
MATH-4715	Introduction to Univalent Functions	3	0	3

Semester-VIII

MATH-4801	Integral Equations	3	0	3
MATH-4802	Capstone Project	3	0	3

Three Optional Courses

MATH-4803	Optimization Theory	3	0	3
MATH-4804	Fluid Mechanics-II	3	0	3
MATH-4805	Mathematical Modeling	3	0	3
MATH-4806	Fractional Calculus	3	0	3
MATH-4807	Algebraic Topology	3	0	3
MATH-4808	Mathematical System Theory	3	0	3
MATH-4809	Dynamical Systems	3	0	3
MATH-4810	Quantum Mechanics-II	3	0	3
MATH-4811	Introduction to Combinatorics	3	0	3
MATH-4812	Variational Inequalities	3	0	3
MATH-4813	Theory of Elasticity	3	0	3

MATH-4814	Electromagnetic Theory-II	3	0	3
MATH-4815	Special Theory of Relativity	3	0	3

2. Scheme of Studies for MPhil in Mathematics: General Breakup

Content		Description	Remarks
2.1	Awarding Institute/Body	Mirpur University of Science and Technology (MUST)	
2.2	Teaching Institute	Department of Mathematics, Mirpur University of Science and Technology (MUST),	
2.3	Final Award	Master of Philosophy in Mathematics	
2.4	Program Title	MPhil in Mathematics	
2.5	Starting Time for Program	Fall/Spring Semester (As per MUST Policy)	
2.6	Duration of the Program	4-8 Semester i. According to HEC 2023 Policy minimum duration reduce to 1.5 years ii. Maximum duration as Per HEC 2023 Policy	
2.7	Eligibility Criteria	BS or MSc or Equivelent Degree in Mathematics with CGPA 2.50 or above (for semester system degree) or 2nd division (for annual system)	
		No D-grade in academic career	
		NTS/ GAT General Test with minimum cumulative score of 50%	
		As per MUST and HEC 2023 Policy	
2.8	Intra-disciplinary Qualifications	As per HEC and MUST Policy 2023	
2.8	Merit Formula	Merit formulas are the following: **For M.Sc:** 15% of Intermediate, 20% of B.Sc, 20% of M.Sc, 35% of Entry Test marks, and 10% of interview conducted by the department. **For BS:** 15% of Intermediate, 40% of BS, 35% of Entry Test marks, and 10% of interview conducted by the department.	
2.9	Total Credit Hours	Course Work: 24 Credit Hrs	
		Seminar: 1 Credit Hrs	
		Thesis (Compulsory): 6 Credit Hrs	

2.10 Program Educational Objectives:

After graduation, our students will be equipped not only with advanced mathematical tools but will also acquire skill set needed to apply mathematics towards engineering problems. Moreover, our graduates will be able to:

i. Collaborate with engineers, scientists and other professionals from industry and academia on research/projects.

ii. Promote the culture of interdisciplinary novel research and produce fundamental & applied quality research in Pakistan

iii. Contribute through publishing fundamental research in the emerging areas of science and engineering, like, systems and control, computational fluid dynamics, mathematical modelling, and complex analysis etc.

2.11 Program Learning Outcomes (PLOs):

The curriculum for MPhil program is so designed that the students undertaking research in this department will have a chance to learn not only the fundamental courses of mathematics but also advanced courses related to their area of specialization and interest. Fundamental and emerging specializations in the domain of mathematics, like systems and control, computational and theoretical fluid dynamics, advanced complex analysis, mathematical modelling of biological systems, and applied and theoretical statistics etc., will be offered as area of research for graduate students at this department. The mathematics department is also providing support to other engineering and sciences departments of MUST, so upon successful completion of the courses taught by mathematics faculty, students will be able to:

PLO-01: Apply knowledge of mathematics, science, and engineering fundamentals to the solution of complex problems involved in different areas of engineering and sciences.

PLO-02: Identify, formulate, search literature, and analyze mathematical models governing laws of physics and other engineering sciences.

PLO-03: Design solution strategy for mathematical models arising in aerospace engineering, electrical engineering, mechanical engineering, and other science and engineering disciplines.

PLO-04: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools, including prediction and modeling the physical phenomena with an understanding of the limitations.

PLO-05: Communicate effectively on mathematical activities with the engineering community and with society at large, such as being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive instructions effectively.

PLO-06: Apply ethical principles and exhibit commitment to professional ethics, responsibilities and norms of the profession.

2.12 Scope of the Program:

The mathematical experts are in demand across all kind of industries, the world over. The curriculum of MPhil mathematics is so designed that students in this department will have a chance to learn not only the fundamental courses of mathematics but also advanced courses and tools related to the emerging areas of applied mathematics like systems and control, mathematical modelling, computational fluid dynamics, complex analysis, statistics, etc. Some of the career opportunities for our graduates are listed in the following.

a). Teaching at school, college, or higher level

b). R&D and strategic organizations like PAEC, NESCOM, SUPARCO, KRL, etc.

c). Banking sector, trading, and stock exchange businesses

d). Higher studies in national and international universities and institutes

e). Armed forces, civil services, oil and gas sector, etc.

2.13 Program Structure and Features, Curriculum Units, Credit and Award Requirements

Sr. No.	Category	No. of Courses	Credit Hrs	Remarks
01	Elective Courses	08	24	
02	Seminar	01	01	
03	Thesis	01	06	
Total			31	

2.14 Semester-Wise Breakdown

Semester-I

Following advanced courses of Mathematics will be offered in this semester.

1. Elective I
2. Elective II
3. Elective III
4. Elective IV

Semester-II

Following foundation courses of the specialized areas will be offered in this semester.

1. Elective V
2. Elective VI
3. Elective VII
4. Elective VIII

Semester-III

Students will start research work in this semester. Students will register in Thesis which will be concluded in Semester IV.

Semester-IV

Students will submit and defend their thesis at the end of this semester. The Seminar will also be graded in this semester. The students who can't finish their thesis by the end of the 4th semester, will seek approval from the relevant authority (AS&RB) for an extension to complete their degree.

2.15 List of Courses for M.Phil Program

A. Compulsory Requirements

Code		Credit Hours	Course Title
MATH-798	Seminar	01	
MATH-799	Thesis	06	

In the following, a list of elective courses is given. A student will have to complete 24 credit hours of his/her course work from these courses.

Code	Course Title	Credit Hours
MATH-701	Advanced Mathematical Physics	03
MATH-702	Advanced Complex Analysis	03
MATH-703	Advanced Topology	03
MATH-704	Advanced Abstract Algebra	03
MATH-705	Advanced Numerical Analysis	03
MATH-706	Advanced Partial Differential Equations	03
MATH-707	Geometric Functions Theory	03
MATH-708	Advanced Optimization Theory	03
MATH-709	Advanced Mathematical Modeling	03
MATH-710	Optimal Control	03
MATH-711	Mathematical Techniques for Boundary Value Problems	03
MATH-712	Non-Newtonian Fluid Mechanics	03
MATH-713	Fundamentals of the Theory of Fluids	03
MATH-714	Group Methods for Differential Equations	03
MATH-715	Fundamentals of Finite Element Methods	03
MATH-716	Advanced Integral Equations	03
MATH-717	Approximation Theory	03
MATH-718	Complex Analysis of Several Variables	03
MATH-719	Advanced Analytical Dynamics	03
MATH-720	Introduction to Robotics	03
MATH-721	Stochastic Processes	03

MATH-722	Estimation Theory	03
MATH-723	Time Series	03
MATH-724	Mathematical Ecology	03
MATH-725	Biomathematics	03
MATH-726	Advances in Discrete Mathematics and Applications	03
MATH-727	Graph Theory	03
MATH-728	Lie Algebra	03
MATH-729	Fuzzy Algebra	03

2.16 Details of the Courses/Contents

Course Code: MATH-701 **Title:** Advanced Mathematical Physics **Credit Hrs:** 03

Course Outline: Nonlinear ordinary differential equations, Linear partial differential equations, classification, initial and boundary values problems, Fourier analysis, Heat equation, Wave equation, Laplace equation etc. Integral equations, classification, d,Alembert solutions, Initial boundary value problems, semi-infinite string with a fixed end, equations with non-homogeneous boundary conditions, vibration of finite string with fixed ends, non-homogeneous wave equations, Method of separation of variables, vibration of a circular membrane: symmetric case, vibration of a circular membrane: general case, non-homogeneous problems, time independent boundary conditions, method of eigen function expansion, Method of eigenfunction using Green's formula, forced vibrating membranes and resonance, methods of Green's function.

Text and Reference Books

1. G. Stephenson and P. M. Radmore, *Advanced Mathematical Methods for Engineeering and Science Students*, Cambridge University Press, 2006.
2. K. T. Tang, *Mathematical Methods for Engineers and Scientists*, Volumes I, II & III, Springer, 2007.
3. M. Stone and P. Goldbart, *Mathematics for Physics*, Academic Press, 2005.
4. S.I. Hayek*, Advanced Mathematical Methods in Science and Engineering*, 2010.

Course Code: MAT-702 **Title:** Advanced Complex Analysis **Credit Hrs:** 03

Course Outline: An Overview of the Theory of a Complex Variables, Argument Principle, Rouche's Theorem, Conformal Mapping, Harwitz Theorems, Cauchy's Theorem on Partial Fraction Expansions, Inverse and Implicit Functions, Riemann Mapping Theorem, Univalent Functions, Basic Properties of Harmonic, Positive Harmonic and Subharminic Functions, Poisson Integral Formula, Mittag-Leffler and Weierstrass Theorems, Infinite Products and Weierstrass Factorisation Theorem, Analytic Continuation, Special Functions Including Gamma Function, Riemann Zeta Function, Prime Number Theorem, Elliptic Functions

Text and Reference Books:

1. L. V. Ahlfors, *Complex Analysis*, M. G. Hill, 1979.
2. R.V. Churchill and J.W. Brown, *Complex Variables and Applications*, 5th Edition, M. Graw Hill, 1989.
3. J. B. Conway, *Functions of one Complex Variable*, Springer, 1979.

4. E. Hill, *Analytic Function Theory*, Vol. I and II, Chelsea Publishing Company, New York, 1974.

Course Code: MATH-703 **Title:** Advanced Topology **Credit Hrs:** 03

Course Outline: Convergence: Sequence and Nets, Filterbase in Space, Convergence, Properties of Filterbases, Closure in Terms of Filterbase, Continuity, Convergence in Cartesian Products, Adequacy of Sequences, Maximal Filterbase

Compactness: Compact Spaces, Special Properties of Compact, Countable Compactness, Compactness in Metric Spaces, Perfect Maps, Local Compactness, O-Compact Spaces, Compactification, K-Space, Baire Space Category

Function Spaces: The Compact Open Topology, Continuity of Composition, The Evaluation Map, Cartesian Products, Application to Identification Topologies, Basis for Z^y, Compact Subsets of Z^y Sequential Convergence in the C-Topology, Metric Topologies, Relation to the C-Topologies, Point-Wise Convergence, Comparison of Topologies in Z^y

The Spaces C(Y): Continuity of the Algebraic Operations, Algebras in C(Y; C), Stone-Weierstrass Theorem, The Metric Space C(Y), Embedding of Y in C(Y), The Ring C(Y).

The Complete Spaces: Cauchy Sequences, Complete Metrics and Complete Spaces, Cauchy Filterbases, Total Boundedness, Baire's Theorem for Complete Metric Spaces, Extension of Uniformly Continues Maps, Fixed Point Theorem for Complete Spaces, Complete Subspaces of Complete Spaces, Complete Gauge Structure.

Text and Reference Books

1. J. Dugundji, *Topology*, Allyn and Bascon, Boston, 1966.
2. J. L. Kelley, *General Topology*, Ishi Press, 2008.
3. J. R. Munkress, *Topology A First Course*, Prentice Inc. Englewood Cliffis, New Jessy, 2nd Edition, 2005.
4. S. Davis, *Topology*, 1st Edition, McGraw-Hill Math, 2004.

Course Code: MATH-704 **Title:** Advanced Abstract Algebra **Credit Hrs:** 03

Course Outline: Endomorphism and Automorphism of Groups, Simple Groups (Definition and Examples), Direct Product of Groups, Sylow Groups and Sylow Theorems, Normal Series of a Group, Refinement Theorem, Composition Series and Jordan Holder Theorem, Solvable and Nilpotent Groups, Finitely Generated Abelian Groups.

Text and Reference Books

1. P. M. Cohn, *Algebra,* Vol. I & II, John Wiley & Sons,
2. S. Lang, *Algebra*, Springer Verlag, 2002.
3. D.S. Dummit and R. M. Foote, *Abstract Algebra*, 3rd Edition, Wiley, 2003.
4. J. A. Gallian, *Contemporary abstract algebra*, D.C. Heath, 1986.

Course Code: MATH-705 **Title:** Advanced Numerical Analysis **Credit Hrs:** 03

Course Outline: Numerical Differentiation, Richardson's Extrapolation, Elements of Numerical Integration, Composite Numerical Integration, Romberg Integration, Adaptive Quadrature Methods, Gaussian Quadrature, Multiple Integrals, Improper Integrals
Discrete Least Squares Approximation, Orthogonal Polynomials and Least Squares Approximation, Chebyshev Polynomials and Economization of Power Series, Rational Function and Trigonometric Polynomial Approximations, Fast Fourier Transforms
Linear Algebra and Eigenvalues, The Power Method, Householder's Method
Fixed Points for Functions of Several Variables, Newton's Method, Quasi-Newton Methods, Steepest Descent Techniques, Homotopy and Continuation Methods
The Linear Shooting Method, The Shooting Method for Nonlinear Problems, Finite-Difference Methods for Linear Problems, Finite-Difference Methods for Nonlinear Problems, The Rayleigh-Ritz Method
Elliptic Partial Differential Equations, Parabolic Partial Differential Equations, Hyperbolic Partial Differential Equations, An Introduction to the Finite-Element Method

Text and Reference Books

1. R. L. Burden and 1. D. Faires, *Numerical Analysis*, 9th Edition, PWS Publishing Company, 2011.
2. K.E. Atkinson, *An Introduction to Numerical Analysis*, 2nd Edition, John Wiley and Sons, New York, 1989.
3. U. Ascher, R. Matteij and R. Russell, *Numerical Solution of BVPs for ODEs*, Prentice Hall, 1988.
4. M. S. Allen, I. Herrea and George F., *Numerical Modeling in Science and Engineering*, Wiley-Interscience, 99 edition, 1988.

Course Code: MATH-707 **Title:** Geometric Functions Theory **Credit Hrs:** 03

Course Outline: Univalent Functions, Some Elementary Transformations of Univalent Functions and Their Properties, Growth and Distortion Theorems for Univalent Functions, Functions with Positive Real Part and Their Related Concepts, Star-Shaped Domains and Starlike Functions, Convex Domains and Convex Functions, Coefficient Bounds, Distortion and Growth Theorem, Functions with Bounded Mocanu Variations
Spirallike, Close-to-Convex and Quasi-Convex Functions, Functions with Bounded Boundary and Bounded Radius Rotations, Bazilevic Functions, Subordination and Convolution in Geometric Function Theory and Their Applications, Some Operators

Text and Reference Books

1. P.L. Duren, *Univalent Functions*, Grundlehren der Math. Wissen chaften, Springer-Verlag, New York-Berlin, 1983.
2. A. W. Goodman, *Univalent Functions*, Vol. I and II, Polygonal Publishing House, Washington, New Jersey, 1983.
3. D. G. Zill and P. D. Shanahan, *A First Course in Complex Analysis with Applications*, Jones and Bartlet Publishers, Sudbury, Massachusetts, 2008.
4. S. G. Krantz, *Explorations in Complex Geometric Function Theory Analysis*, 2006 Edition, Birkhäuser, 2005.

Course Code: MATH-708 **Title:** Advanced Optimization Theory **Credit Hrs:** 03

Course Outline: Intro to mathematical optimization, objective of constraint functions, basic optimization concepts, mathematical prerequisites, convexity, gradient vectors, Hessian matrix, global and local minima, saddle points, optimality conductions, general structure for line search

method, Wolf conditions, Goldstein conditions, convergence of line search method, convergence of steepest decent method, Newton method, Quasi Newton method, Newton method with Hessian approximation, line search algorithm for Wolf conditions, conjugate gradient method, Trust Region Method and Cauchy point, Newton point, Dogleg method.

Text and Reference Books

1. L. Elsgolts, *DEs and the Calculus of Variations*, Mir Publishers Moscow, 1970.
2. B. S. Gotfried and J. Weisman, *Introduction to Optimization Theory*, Prentice Hall, Englewood Cliffs, NJ, USA, 1973.
3. D. G. Luenberger, *Introduction to Linear and Non-Linear Programming*, Addision Wesley, Reading, USA, 1973.
4. D. A. Pierre, *Optimization Theory with Applications*, Dover Publications, 1986.

Course Code: MATH-709 **Title:** Advanced Mathematical Modeling **Credit Hrs:** 03

Course Outline: Modeling through Differential Equations, Mathematical modeling through Ordinary Differential Equations, Mathematical modeling through system of Ordinary Differential equations of first order, Excursive of some modeling projects to demonstrate the variety of the require equations to formulate essential, control theory as modeling tool, Parameter estimation as optimization problem, Sensitivity analysis, Parameter estimation, Situation giving rise to Partial Differential Equations, Modeling through Delay differential and Differential–Difference equations

Text and Reference Books

1. Edwards, D. and Hamson, *Mathematical modeling Skills*, Macmillan Press Ltd, 1996.
2. Giordano, F.R., Weir, M.D. and Fox, W.P., *A First Course in Mathematical modeling* , Thomson Brooks/Cole, 2003.
3. J.N.Kapur, *Mathematical modeling*, New Age international (P) Limited, 2015.
4. M. M. Meerschaert, *Mathematical modeling*, 4th Edition, Academic Press, 2013.

Course Code: MATH-710 **Title:** Optimal Control **Credit Hrs:** 03

Course Outline: Calculus of Variation: Principle of calculus of variation, Euler Equation in Calculus of variation, some important theoretical results of calculus of variation and mathematical examples

Optimal Control and the Minimum Principle: Application of calculus of variation in optimal control, simplest problem in the Calculus of variation, Necessary condition for optimality, Lagrange multipliers, and Hamiltonian equations, The minimum principle, solution of Zermelo's problem, Linear quadratic cost problem with finite and infinite horizon: theory and examples,

Optimal Control Theory and Dynamic Programming: Dynamic programming in discrete time, Principle of optimality and Bellman's equation, linear quadratic cost problem with finite and infinite horizon in the dynamic programming perspective: theory and examples,

Differential Games: Introduction, continuous time differential games, intro to Nash and Stackleberg equilibrium solutions concepts, solutions of Nash differential games: theory and examples

Text and Reference Books

1. D. Liberzon, *Calculus of Variations and Optimal Control Theory:* A Concise Introduction, Princeton University Press, 2012
2. G. Meisma, *Lecture Notes on Optimal Control*, University of Twente, Enschede,The Netherlands, 2013.
3. D. E. Krik, *Optimal Control Theory*: An Introduction, Dover Publications, 2004.
4. R. F. Stengel, *Optimal Control*, Dover Publications, 1994.

Course Code: MATH-711 **Title:** Mathematical Techniques for BVPs **Credit Hrs:** 03

Course Outline: Green's Function Method, Perturbation Method: Regular and Singular Perturbation Techniques with Application, Variational Methods,Wiener -Hopf Techniques with Applications to Diffraction Problems.

Text and Reference Books

1. R. Mitra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves*, Macmillan, 1971.
2. E. J. Hinch, *Perturbation Methods*, Cambridge University Press, 1991.
3. B. Noble, *Methods Based on the Wiener Hopf Techniques for the Solution of Partial Differential Equations*, 2nd Edition, American Mathematical Society, 1988.
4. I. Stakgold, *Boundary Value Problems of Mathematical Physi*cs, Society for Industrial and Applied Mathematics, 1987.

Course Code: MATH-712 **Title:** Non-Newtonian Fluid Mechanics **Credit Hrs:** 03

Course Outline: Classification of Non-Newtonian Fluids, Rheological Formulates (Time–Independent Fluids, Thyrotrophic Fluids and Viscoelastic Fluids), Variable Viscosity Fluids. Cross Viscosity Fluids, The Deformation Rate, The Rivlin-Ericksen Fluid, Basic Equation of Memories in Rheological Models. The Linear Viscoelastic Liquid, Couette Flow, Poiseuille Flows, The Current Semi-Infinite Field. Axial Oscillatory Tube Flow, Angular Oscillatory Motion, Periodic Transients, Basic Equations in Boundary Layer Theory, Orders of Magnitude, Truncated Solutions for Viscoelastic Flow, Similarity Solutions. Turbulent Boundary Layers, Stability Analysis.

Text and Reference Books

1. G. Astarita and M. G, .*Principles of Non-Newtonian Fluid mechanics*, McGraw-Hill 1974.
2. R. B. Bird, R. C. Armstrong and O. Hassager, *Dynamic of Polymeric liquids*,Vol. 1 and II, John Wiley and Sons, New York, 1987.
3. W. F. Robert and A. T. McDonald, *Introduction to Fluid Mechanics*, John Wiley and Sons, 1995.
4. W. R. Schowalter, *Mechanics of Non-Newtonian Fluids*, Pergamon Press, New York, 1978.

Course Code: MATH-713 **Title:** Fundamentals of the Theory of Fluids **Credit Hrs:** 03

Course Outline: Euler's Equations of Motion, Viscosity, Navier-Stokes Equations and Exact Solutions, Dynamical Similarity and Reynolds Number, Energy Equation, Boundary Layer Concept and Governing Equations, Magnetohydrodynamics (MHD) Equations, Fluid Drifts, Stability and Equilibrium Problems, Introduction to Turbulent Flow.

Text and Reference Books

1. G. Astarita and G. Merrucci, *Principles of Non-Newtonian Fluid mechanics*, M. Graw Hill, 1974.
2. R. B. Bird, R. C. Armstrong and O. Hassager, *Dynamic of Polymeric liquids*, Vol. I, 2nd Edition, John Wiley and Sons, New York, 1987.
3. J. Harris, *Rheology and Non-Newtonian Flow*, Longman Inc., New York, 1977.
4. Schowalter, *Mechanics of Non-Newtonian Fluids*, Pergamon Press, New York, 1978.

Course Code: MATH-714 **Title:** Group Methods For Differential Equations **Credit Hrs:** 03

Course Outline: Basic concept of groups of transformation, Parameter Lie group of transformation (LGT), infinitesimal transformation (I.T), Infinitesimal generators, Lie's first fundamental theorem, Invariance, Canonical coordinates, Elongations, Multi-parameter Lie group of transformation (MLGT), Lie algebra, Solvable Lie algebra, Lie's second and third fundamental theorems.

Invariance of ODE's under (LGT) and (MLGT), Mapping solutions to other solutions from invariance of an ODE and PDE, Determining equations (I.T) of and n^{th} order ODE and system of PDE's, Determination of n^{th} order ODE invariant under a given group, Reduction of order by canonical coordinates and differential invariants, invariant solutions of ODE's and PDE's Separatices and envelops. Neother's theorem and Lie-Backlund symmetries, Potential symmetries, Mapping of differential equations.

Text and Reference Books:

1. J. M. Hill, *Differential Equations and Group Methods*, CRC Press, Inc. New York, 1992.
2. L. P. Eisnhart, *Continuous Group of Transformation*, Dover Publication, Inc. New York, 1961.
3. P. E. Luther, *Continuous Group of Transformation*, Andesite Press, 2017.
4. P. E. Hyden, *Symmetries and Differential Equations*, 1st edition, Cambridge University Press, 2000.

Course Code: MATH-715 **Title:** Fundamentals of Finite Element Methods **Credit Hrs:** 03

Course Outline: Finite Element Methods: General Approach, The Galarkin Method in One and More Dimensions and Application, Error Bound on the Galarkin Method and Application, The Method of Collocation, Error Bounds on the Collocation Method and Application, Comparison of Efficiency of the Finite Difference and Finite Element Method, Application to Solution of Linear and Non-Linear Partial Differential Equations Appearing in Physical Problems

Text and Reference Books

1. S. D. Burnett, Finite Element Analysis from Concept to Applications, Addison Wesley, 1987.
2. G. S. Desai, *Elementary Finite Element Metho*d, Prentice Hall, 1988.
3. G. Strang and G. Fix, *Analysis of Finite Element Method*, Prentice Hall, New Jersey, 1973.
4. J. N. Reddy, *Introduction to the Finite Element Method*, 3rd Edition, MHI, 2006.

Course Code: MATH-716 **Title**: Advanced Integral Equations **Credit Hrs:** 03

Course Outline: Existence Theorem, Integral Equations with L_2 Kernal, Applications to Partial differential equations, Integral Transforms, Wiener-Hopt Techniques.

Text and Reference Books

1. H. H. Stadl, *Integral Equactions*, John Wiley and Sons, 1973.
2. I. Stakgold, *Boundary Value Problems of Mathematical Physics*, Macmillan, New York, 1968.
3. F. G. Tricomi, *Integral Equations*, Inter science, 1957.
4. J. J. Abdul, *Introduction to Integral Equations with Applications,* Marcel Dekker Inc. New York, 1985.

Course Code: MATH-717 **Title:** Approximation Theory **Credit Hrs:** 03

Course Outline: Best Approximation in Metric and Normed Spaces, Least Square Approximation, Rational Approximation, Haar Condition and Best Approximation in Function Spaces, Interpolation, Stone-Weierstrass Theorem for Scalar and Vector-Valued Functions, Spline Approximation, Best Approximations in Normal Spaces, Existence Theorem, Approximation by Algebraic Polynomials, Uniqueness, Strict Convexity, Lemma Convexity, Uniqueness Theorem, Hilbert Space, Uniform Approximation, External Point, Haar Condition, Extreme Points, Best Approximation, Characterization of Best Approximation, A Brief Introduction to Interpolation, Chebyshev Polynomials, Chebyshev Theorem, Least Squares Approximation, Gram-Schmidt Orthonormalization Process, Orthogonal Function, Pade Approximation, Remex Algorithm

Text and Reference Books

1. N. I. Achieser, *Theory of approximation*, Dover Publications, Inc., New York, 2004.
2. T. J. Rivlin, *An Introduction to the Approximation of Functions*, Dover Publications, Inc., New York, 1981.
3. M. J. D. Powell, *Approximation Theory and Methods,* Cambridge University Press, 1988.
4. E. W. Cheney , *Introduction to Approximation Theory*, AMS, 2000.

Course Code: MATH-718 **Title:** Complex Analysis of Several Variables **Credit Hrs:** 03

Course Outline: Holomorphic Functions: Review of One Variable Theory, Real and Complex Differentiability, Power Series, Complex Differentiable Functions, Cauchy Inequalities, The Maximum Principles.

Extension of Analytical Functions: Hartogs Figures, Hortogs Theorems, Domains of Holomorphy, Holomorphic Convexity, Theorem of Cartan Thullen.

Levi-Convexity: The Levi Forms, Geometric Interpretation of its Signature, E. E. Levi's Theorem, Connections with Khlerian Geometry, Elementary Properties of Plurisubharmonic Functions.

Introduction to Cohomology: Definition and Example of Complex Manifolds, The D Đ Б Operators, The Poincare Lemma and the Dolbeaut Lemma, The Cousin Problems, Introduction to Sheaf Theory.

Text and Reference Books

1. J. Morrow and K. Kodaria, Complex Manifolds, Holt, Rinehart and Winston, New York, 1971.
2. L. Hormander, *An Introduction to Complex Analysis in Several Variables*, D. V. Nostarnad, New York, 1966.
3. H. Grauert and K. Fritsch Several, *Complex Variables*, Springer Verlag, 1976.
4. M. Field, *Several Variables and Complex Manifolds*, Cambridge University Press, 1982.

Course Code: MATH-719 **Title:** Advanced Analytical Dynamics **Credit Hrs:** 03

Course Outline: Equations of Dynamic and its Various Forms, Equations of Langrange and Euler, Jacobi's Elliptic Functions and the Quantitative and Quantitative Solutions of the Problem of the Euler and Poisson, the Problems of Langrange and Poisson, Dynamical System, Equations of Hamilton and Appell, Hamilton–Jacobi Theorem, Separable System, Holder's Variational Principle and its Consequences. Group of Continuous Transformations and Poincare's Equations, System with one Degree of Freedom, Singular Points, Cyclic Characteristics of System With N-Degree of Freedom, Ergodic Theorem, Metric Indecompossability, Stability of Motion, Periodic Orbits.

Text and Reference Books

1. H. Baruh, *Analytical Dynamics*, Heinmann, 1st Edition, WCB/McGraw-Hill, 1998.
2. E. T. Whittaker, *A treastise on Dynamic of Rigid Bodies and Particles*, At the University Press, 1927.
3. V. D. Sapio, *Advanced Analytical Dynamics: Theory and Applications*, 1st Edition, Cambridge University Press, 2017.
4. L. Meirovitch, *Methods of analytical dynamics*, McGraw-Hill , 1970.

Course Code: MATH-720 **Title:** Introduction to Robotics **Credit Hrs:** 03

Course Outline: Fundamental Concepts: Introduction to Robot (Fundamental notions and Definitions), **Jacobians:** Transformations and Jacobians, Manipulator. **Kinematics:** Kinematics (Forward and Inverse) of manipulator, Manipulator Dynamics, Trajectory Generation, Manipulator Mechanism, Manipulator Design. **Linear Control:** Linear Control of Minipulator, Non-linear Control of Manipulator, Forced Control of Manipulator, **Multivariable Control:** Multivariable control, Feedback linearization, Variable structure and Adaptive Control.

Text and Reference Books

1. J. J. Craig, *Introduction to Robotics, Addison-Wesley Publishing Company, 1999.*
2. Mark, W. Sponge, and M. Vidyasagar, *Robot Dynamics Control*, John Wiley and Sons, 2004.
3. G. Franklin, and J. D. Powell, *Feed-back Control of Dynamic Systems, Addison-Wesley Publishing Company, 1989.*
4. *S. M. Shinners, Modern Control System Theory and Applications, Addison-Wesley Publishing Company, 1987.*

Course Code: MATH-721 **Title:** Stochastics Processes **Credit Hrs:** 03

Course Outline: Definition of a Stochastic Process, Characterization of a Stochastic Process, Discrete-Time and Continuous-Time Stochastic Process and Their First and Second Order Statistics, Continuity, Derivative and Integral of a Stochastic Process, Time-Averages and Ergodic Theorems,

Stationarity, Power Spectral Density, Time- Series Analysis, Discrete-Time Markov Chains, Continuous-Time Markov Chains and Introduction To Queuing Theory

Text and Reference Books

1. W. Whitt, Stochastic-Process Limits: *An Introduction to Stochastic-Process Limits and Their Applications to Queue*s, Springer, New York, USA, 2002.
2. Z. Brzeźniak and T. Zastawniak, *Basic Stochastic Processes: A Course Through Exercises,* Springer, NY, 2005.
3. S. M. Ross, *Stochastic-Process*, 2nd Edition, Wiley, 1995.
4. R. G. Gallager, *Stochastic-Process*, 1st Edition, Cambridge University Press, 2014.

Course Code: MATH-722 **Title:** Estimation Theory **Credit Hrs:** 03

Course Outline: Properties of Estimator, Unbiaseness, Efficiency, Sufficiency and the minimal sufficient statistic, complete classes, Exponential families, Cramer-Rao lower bound and its extension, bias reduction by Jackknifing, ancilliary and Basu theorem, methods of estimation and their optimal properties, Bayes and minimax estimators, shrinkage estimation, sequential estimation. Non Linear Models, Parameters and estimation using ML method, Transformations of parameters, inference and stable transformations. Computing Methods for Non-linear Modelling, Confidence intervals for parameters and functions. Applications of non-linear modelling.

Text and Reference Books

1. Levy, P. S., and Lemeshow, S., *Sampling of Populations Methods and Applications*, 3rd Ed. John Wiley, New York (1999)
2. Lindgren, B. W., *Statistical Theory*, Chapman and Hall (1998).
3. Lehman, E. L., *Theory of Point Estimation*, John Wiley and sons (1987).
4. Ross, G. J. S., *Non-linear Estimation*, Springer-Verlag, New York Inc., (1990).

Course Code: MATH-723 **Title**: Time Series **Credit Hrs:** 03

Course Outline: Methods of decomposing time-series into its component parts and measuring their effects. Stationary stochastic processes and their properties in the time domain. Frequency domain-Cyclical trend, spectral representation of a stochastic process. Properties of ARMA process and linear filters. Multivariate spectral analysis. Estimation of ARMA models. Model building and forecasting. Practicals based on the above topics

Text and Reference Books

1. A. C. Harvey, *Time Series Models*, Philip Allan Publishers, 1981.
2. I. G. Zurbenko, *The Spectral Analysis of Time Series*, Elsevier Science Publishers B.V., Amsterdam, 1986.
3. E. J. Hannan, *Time Series Analysis*, Methuen, London, 1960.
4. M. G. Kendall, *Time-Series*, Griffin, London, 1973.

Course Code: MATH-724 **Mathematical Ecology** **Credit Hrs: 3**

Course Outlines: Key models in Behavioral Ecology, Diet-choice and foraging, Evolutionarily Stable Strategies, Search and Predation, Stochastic models and statistics, Probability background and important distributions, Some applications to search and foraging, Bayesian methods, Host-parasitoid models, Nicholson-Bailey and extensions, Evolutionary models and stochastic dynamic programming, Disease models and Fishery models (may be included based on participant interest and

available time), Basic SIR and extensions, Evolution of virulence, Vectors and disease, Fisheries bio-economic models, Stochastic population models, Sample paths and stochastic differential equations, General stochastic diffusion processes, Extinction time in density independent case, Extinction time in density-dependent case, Designing a model Cellular automata and IBMs, Formulating and implementing a model

Text and Reference Books:

1. Allen, L. J. S., *An Introduction to Stochastic Processes with Applications to Biology*. Pearson. Upper Saddle River, NJ, USA, 2003.
2. Allen, L. J. S., *An Introduction to Mathematical Biology*. Pearson. Upper Saddle River, NJ, USA, 2007.
3. Allman, E. S. and J. Rhodes., *Mathematical Models in Biology*: An Introduction. Cambridge University Press, 2004.
4. Brauer, F. and C. Castillo-Chavez., *Mathematical Models in Population Biology and Epidemiology*, Springer, NY, 2001.

Course Code: MATH- 725 **Biomathematics** **Credit Hrs: 3**

Course Outlines: Biological applications of difference and differential equations, Biological applications of nonlinear differential equations, Biological applications of graph theory.

Text and Reference Books:

1. Linda J. S. Allen, *An introduction to Mathematical Biology*, Pesrson, 2007.
2. J. Smith, *Mathematical ideas in biology*, Cambridge Press, Cambridge, 1968.
3. S. I. Rubinow, *An introduction to Mathematical Biology*, 1st edition, Dover Publications, 2003.
4. B. P. ingalls, *Mathematical Modeling in Systems Biology*, 1st edition, The MIT Press, 2013.

Course Code: MATH-726. **Advances in Discrete Mathematics and Applications Credit Hrs: 3**

Course Outlines: Introduction, Definitions of stability and linearized stability, Semi-cycle analysis, Full limiting sequences and Convergence theorems, Lyness equation, Todd equation, the generalized Lozi equation, the Gingerbreadman equation and the Riccati difference equation, analysis of semi-cycle of some special type of equations, Period-2 solutions, Global asymptotic stability of period-2 solutions, Existence of unbounded solutions and Boundedness of solutions**,** on the systems of rational difference equations

Text and Reference Books:

1. M. R. S. Kulenovic, *Dynamics of Second Order Rational Difference Equations: With Open Problems and Conjectures*, Chapman and Hall/CRC, 2005.
2. E. Camouzis, G. Ladas, *Dynamics of Third-Order Rational Difference Equations with Open Problems and Conjectures*, Chapman and Hall/CRC, 2007.
3. M. H. Education, *Advanced Mathematical Concepts*, 6th Edition, McGraw-Hill Education, 2003.
4. K. Abidi and J. X. Xu, *Advanced Discrete-Time Control: Designs and Applications*, 5th Edition, Springer, 2015.

Course Code: MATH-727 **Title:** Graph Theory **Credit Hrs:** 03

Course Outline: Undirected Graphs, Geometric Graphs, Abstract Graphs, Isomorphism, Edge Progressions Chains and Circuits, Rank and Nullity, Degrees, Trees, Bipartite Graphs, Unicursal Graphs, Hamiltonian Graphs. Directed Graphs, Arc Progressions, Paths Progression and Cycle Progression

Partition and Distances in Graphs, Edge Partitions, Arc Partitions, Hamiltonian Chains and Circuits, Vertex Partitions, Radius and Diameter, Minimal Length Problem, Foundation of Electrical Network Theory, Matrix Representation, The Incidence Matrix, The Circuit Matrix, The Cut-Set Matrix, The Vertex or Adjacency Matrix, The Path Matrix, Network Flows, Network Flow Problems.

Text and Reference Books

1. J. A. Bondy and U. S. R. Murty, *Graph Theory*, Springer Verlag, York, 2008.
2. B. W. Douglas, *Introduction to Graph Theory*, Prentice Hall, 2001.
3. G. Jonathan and Y. Jay, *Graph Theory and its Applications*, CRC Press, London 1999.
4. R. J. Wilson, *Introduction to Graph Theory*, Prentice Hall Wesley, 1996.

Course Code: MATH-728 **Title:** Lie Algebra **Credit Hrs:** 03

Course Outline: Definitions and Example of Lie Algebras, Ideals and Quotients, Simple, Solvable and Nilpotent Lie Algebras, Radial of a Lie Algebra, Semi-simple Lie Algebra, Engel's Nil Potency Criterion, Lie's and Cartan Theorems, Jordan-Chevalley Decomposition, Killing Forms, Criterion for Semisimplecity, Product of Lie Algebra, Classification of Lie Algebra up to Four Dimension, Application of Lie Algebras.

Text and Reference Books

1. J. E. Humphreya, *Introduction to the Algebra and Representation Theory*, S. Verlag, 1972.
2. N. Jacobson, *Lie Algebr*a, Intersience, New York, 1983.
3. J. Lapowsky and G.W. Macculum, *Elementary Lie Algebra Theory*, Yale University, 1974.
4. O. Neill, *Semi-Riemannian Geometry*, Academic Press, 1983.

Course Code: MATH-729 **Title:** Fuzzy Algebra **Credit Hrs:** 03

Course Outline: Introduction, The Concept of Fuzziness Examples, Mathematical Modeling, Operations of fuzzy sets, Fuzziness asuncertainty. **Algebra of Fuzzy Sets:** Boolean Algebra and lattices, Equivalence relations and partions, Composing mappings, Alpha-cuts, Images of alpha-level sets, Operations on fuzzy sets. **Fuzzy Relations:** Definition and examples, Binary Fuzzy relations Operations on Fuzzy relations, fuzzy partitions. **Fuzzy Semigroups:** Fuzzy ideals of semigroups, Fuzzy quasi-ideals, Fuzzy bi-ideals of Semigroups, Characterization of different classes of semigroups by the properties of their fuzzy ideals fuzzy quasi-ideals and fuzzy bi-ideals. **Fuzzy Rings:** Fuzzy ideals of rings, Prime, semiprime fuzzy ideals, Characterization of rings using the properties of fuzzy ideals

Text and Reference Books

1. Hung T. Nguyen and A First course in Fuzzy Logic, Chapman and Hall/CRC Elbert A. Walker 1999.
2. M. Ganesh, Introduction to Fuzzy Sets and Fuzzy Logic, Prentice-Hall of India, 2006.
3. John N. Mordeson and Fuzzy Commutative algebra, World Scientific, 1998.D.S. Malik,
4. John N. Mordeson, Fuzzy Semigroups, Springer-Verlage, 2003. D.S. Malik and Nobuki Kuroki

3. Scheme of Studies for PhD in Mathematics: General Breakup

Content		Description	Remarks
3.1	Awarding Institute/Body	Mirpur University of Science and Technology (MUST)	
3.2	Teaching Institute	Department of Mathematics, Mirpur University of Science and Technology (MUST)	
3.3	Final Award	Doctor of Philosophy in Mathematics	
3.4	Program Title	PhD in Mathematics	
3.5	Starting Time for Program	Fall/Spring Semester of Every Academic Year	
3.6	Duration of the Program	3 to 8 years	
3.7	Entrance Requirement	MPhil or MS or Equivalent Degree in Mathematics with CGPA 3.0 or above (for semester system degree) or 1st division (for annual system). As per HEC and MUST Policy	
		No D-grade in academic career	
		The GRE subject test with a minimum of 60% (for admissions thereafter) percentile score is required.	
3.8	Merit Formula	Merit formulas are the following: **For M.Sc:** 15% of Intermediate, 20% of B.Sc, 20% of M.Sc, 25% of M.Phil marks, 5% Publications, and 15% of interview conducted by the department. **For BS:** 15% of Intermediate, 40% of BS, 25% of M.Phil marks, 5% Publications, and 15% of interview conducted by the department.	
3.9	Total Credit Hours	Course Work: 18 Credit Hrs	
		Thesis (MAT-899): 09 Credit Hrs	
		Seminar I & II (MAT-897, 898): 1 Credit Hrs each	
		Comprehensive Examination (Written and Oral): P/F	HEC Policy 2023

3.10 Program Educational Objectives:

Our students will be equipped not only with advanced mathematical tools but will also acquire skill set needed to apply mathematics towards real life problems in different fields of engineering, sciences, economics, business and finance, etc. Moreover, our graduates will be able to:

i. Collaborate with Engineers and scientists from industry and academia in their research/projects to promote the industry-academia linkages;
ii. Promote the culture of interdisciplinary novel research and produce fundamental & applied quality research in Pakistan;
iii. Contribute through active research in the emerging areas of science and engineering, for instance, systems and control, computational and theoretical fluid dynamics,

advanced complex analysis, mathematical modelling of biological systems, and statistics etc.

3.11 Program Learning Outcomes (PLOs):

The curriculum for Ph.D program is so designed that the students undertaking research in this department will have a chance to learn not only the fundamental courses of mathematics but also advanced courses related to their area of specialization and interest. Fundamental and emerging specializations in the domain of mathematics, like systems and control, computational and theoretical fluid dynamics, advanced complex analysis, mathematical modelling of biological systems, and applied and theoretical statistics etc., will be offered as area of research for graduate students at this department. The mathematics department is also providing support to other engineering and sciences departments of MUST, so upon successful completion of the courses taught by mathematics faculty, students will be able to:

PLO-01: Apply knowledge of mathematics, science, and engineering fundamentals to the solution of complex problems involved in different areas of engineering and secinces.

PLO-02: Identify, formulate, search literature, and analyze mathematical models governing laws of physics and other engineering sciences.

PLO-03: Design solution strategy for mathematical models arising in aerospace engineering, electrical engineering, mechanical engineering, and other science and engineering disciplines.

PLO-04: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools, including prediction and modeling the physical phenomena with an understanding of the limitations.

PLO-05: Communicate effectively on mathematical activities with the engineering community and with society at large, such as being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive instructions effectively.

PLO-06: Apply ethical principles and exhibit committment to professional ethics, responsibilities and norms of the profession.

4.12 Scope of the Program: The mathematical experts are in demand across all kind of industries, the world over. The curriculum of MPhil mathematics is so designed that students in this department will have a chance to learn not only the fundamental courses of mathematics but also advanced courses and tools related to the emerging areas of applied mathematics like systems and control, mathematical modelling, computational fluid dynamics, complex analysis, statistics, etc. Some of the career opportunities for our graduates are listed in the following

a). Teaching at school, college, or higher level

b). R&D and strategic organizations like PAEC, NESCOM, SUPARCO, KRL, etc.

c). Banking sector, trading, and stock exchange businesses

d). Higher studies in national and international universities and institutes

e). Armed forces, civil services, oil and gas sector, etc.

3.12 Program Structure and Features, Curriculum Units, Credit and Award Requirements

Sr. No.	Category	No. of Courses	Credit Hrs	Remarks
01	Elective Courses	06	18	
02	Seminar	02	02	
03	Thesis	01	09	
Total			29	

4.13 Semester-Wise Breakdown

Semester-I

Following advanced courses of Mathematics will be offered in this semester.

1. Elective I
2. Elective II
3. Elective III

Semester-II

Following foundation courses of the specialized areas will be offered in this semester.

1. Elective IV
2. Elective V
3. Elective VI

Semester-III and IV:

Students will complete his/her course work (if it is not finished in the first two semesters) and clear the Comprehensive Examination (Written and Oral) by the end of the 4^{th} semester.

Semester-V and VI:

Students will submit synopsis in the 5^{th} semester to the relevant body and after approval of the synopsis, research work will start formally. The two Seminars will be evaluated after the submission of synopsis and before the public defense of the thesis. After completing the research work and the degree award requirements, the process for the public defense will be started. The students who can't finish their thesis by the end of the 6^{th} semester, will seek approval from the relevant authority (AS&RB) for an extension to complete their degree.

3.13 List of Courses for Ph.D Program

A. Compulsory Requirements

Code	Title	Credit Hours
MATH-897	Seminar I	01
MATH-898	Seminar II	01
MATH-899	Thesis	09
MATH-896	Comprehensive Examination (Written & Oral)	P/F

Elective Courses* (18 credit hrs)

In the following, a list of elective courses are given. A student will have to complete 18 credit hours of his/her course work from these courses on the recommendation of his/her supervisor.

Code	Course Title	Credit Hours
MATH-801	Mathematical Analysis	03
MATH-802	Riemanian Geometry	03
MATH-803	Mathematical Techniques	03
MATH-804	Advanced Abstract Algebra	03
MATH-805	Finite Element Methods	03
MATH-806	Advanced Partial Differential Equations	03
MATH-807	Advanced Functional Analysis	03
MATH-808	Variational Inequalities	03
MATH-809	Convex Analysis	03
MATH-810	Parameter Estimation and Sensitivity Analysis	03
MATH-811	Semigroups in Geometric Functions Theory	03
MATH-812	Differential Subordination Theory and Applications	03
MATH-813	Conformal Mappings	03
MATH-814	Perturbation Methods	03
MATH-815	Electro-dynamics	03
MATH-816	Magneto-hydro-dynamics	03
MATH-817	Fundamentals of Turbulence	03
MATH-818	Lie Group Analysis of Differential Equations	03
MATH-819	Selected Topics in Applied Mathematics	03
MATH-820	Selected Topics in Pure Mathematics	03
MATH-821	Numerical Solutions of PDEs	03
MATH-822	Design Methods for Control Systems	03
MATH-823	Optimal State Estimation	03
MATH-824	Linear Matrix Inequalities	03
MATH-825	Stochastic Differential Equations	03
MATH-826	Fixed Point Theory and Applications	03

MATH-827	Integral Inequalities	03
MATH-828	Banach Algebras	03
MATH-829	Harmonic Functions Theory	03
MATH-830	Cosmology	03
MATH-831	Bifurcation and Chaos	03
MATH-832	Nonlinear Systems and Control	03

3.14: Details of the Courses/Contents

Course Code: MATH-801 **Title:** Advanced Mathematical Analysis **Credit Hrs:** 03

Course Outline: Introduction to Fourier analysis: Lebesgue measure, Fubini Theorem, Convolutions, Introduction to the Fourier Transform, **Introduction to Function spaces:** Introduction to Sobolev spaces and distributions, Foundations of Harmonic analysis and theory of maximal operators, Lebesgue points, **Introduction to Geometric measure theory:** Hausdorff metric, Hausdorff measure, Fractals, Hausdorff dimensions, Box dimension,

Text and Reference Books:

1. E. B. Davis, *Spectral Theory and Differential Operators*, Graduate text, Cambridge University Press, 1995.
2. E. H. Lieb and M. Loss, *Analysis, Graduate Studies in Mathematics*, Volume 14, AMS, 1997.
3. V. Maziya and S. Poborchi, *Differntial Functions on bad domains*, World Scientific, 1997.
4. Steven R. L., *Analysis: With an Introduction to Proof*, 4th edition, Pearson, 2004.

Course Code: MATH-802 **Title:** Riemanian Geometry **Credit Hrs:** 03

Course Outline: Definition and examples of manifolds. Differential maps. Submanifolds. Tangents. Coordinate vector fields. Tangent spaces. Dual spaces. Multilinear functions. Algebra of tensors. Vector fields. Tensor fields. Integral curves. Flows. Lie derivatives. Brackets. Differential forms. Introduction to integration theory on manifolds. Riemannian and semi Riemannian metrics. Flat spaces. Affine connection. Parallel translations. Covariant differentiation of tensor fields. Curvature and Torsion tensors. Connection of a semi-Riemannian tensor. Killing equation and Killing vector fields. Geodesics. Conformal transformations and the Weyl tensor.

Text and Reference Books:

5. M. P. do Carmo, Riemannian Geometry, Birkhauser, Boston, 1992.
6. D. Langwitz, Differential and Riemannian Geometry, Academic Press, 1970.
7. R. L. Bishop and S. I. Goldberg, Tensor Nalysis and Manifolds, Dover Publication, Inc., 1980.
8. M. Berger , A Panoramic View of Riemannian Geometry, Springer, 2007.

Course Code: MATH-803 **Title:** Mathematical Techniques **Credit Hrs:** 03

Course Outline: Green's function method with applications to wave-propagation. Solution of algebraic equations by perturbation methods. Evaluation of integrals by expansion of integrands. Laplace methods. The method of stationary phase. The methods of steepest descent. Solution of the linear damped oscillator equation by perturbation methods. The WKB approximation. Variational problems with variable end points. Corner conditions. Sufficient conditions for minimum. The Ritz method and its applications. A survey of transform techniques. Wiener-Hopf technique with applications to diffraction problems.

Text and Reference Books:

1. A. Nayfeh, *Perturbation Methods*, John Wiley & Sons, Inc., 1973.

2. I. Stakgold, *Boundary Value Problems of Mathematical Physics*, Volume I & II, The MacMillan Company, 1970.
3. B. Boble, *Methods Based on the Wiener-Hopf Technique for the Solution of Partial Differential Equations*, Pergamon Press, 1958.
4. J. Jordan and P. Smith, *Mathematical Techniques: An Introduction for the Engineering, Physical, and Mathematical Sciences*, Oxford University Press, 2008.

Course Code: MAT-804 **Title:** Advanced Abstract Algebra **Credit Hrs:** 03

Course Outline: Endomorphism and Automorphism of Groups, Simple Groups (Definition and Examples), Direct Product of Groups, Sylow Groups and Sylow Theorems, Normal Series of a Group, Refinement Theorem, Composition Series and Jordan Holder Theorem, Solvable and Nilpotent Groups, Finitely Generated Abelian Groups.

Text and Reference Books

1. P. M. Cohn, *Algebra*, Vol. I & II, John Wiley & Sons,
2. S. Lang, *Algebra*, Springer Verlag, 2002.
3. D.S. Dummit and R. M. Foote, *Abstract Algebra*, 3rd Edition, Wiley, 2003.
4. J. A. Gallian, *Contemporary abstract algebra*, D.C. Heath, 1986.

Course Code: MATH-805 **Title:** Finite Element Methods **Credit Hrs:** 03

Course Outline: Finite Element Methods: General Approach, The Galarkin Method in One and More Dimensions and Application, Error Bound on the Galarkin Method and Application, The Method of Collocation, Error Bounds on the Collocation Method and Application, Comparison of Efficiency of the Finite Difference and Finite Element Method, Application to Solution of Linear and Non-Linear Partial Differential Equations Appearing in Physical Problems

Text and Reference Books

5. S. D. Burnett, Finite Element Analysis from Concept to Applications, Addison Wesley, 1987.
6. G. S. Desai, *Elementary Finite Element Metho*d, Prentice Hall, 1988.
7. G. Strang and G. Fix, *Analysis of Finite Element Method*, Prentice Hall, New Jersey, 1973.
8. J. N. Reddy, *Introduction to the Finite Element Method*, 3rd Edition, MHI, 2006.

Course Code: MATH-806 **Title:** Advanced Functional Analysis **Credit Hrs:** 03

Course Outline: Normed Linear Spaces, Bounded Linear Transformation, Finite Dimensional Normed Linear Spaces, Some Dual Spaces, The Hahn Banach Theorem, The Algebra of Bounded Linear Operators, Baire's Theorem, The Principle of Uniform Boundedness, The Open Mapping Theorem, The Closed Graph Theorem, Hilbert Spaces, Spectral Theory in Hilbert Spaces, Hermitian Symmetric Forms, Orthogonality Self Adjoint Bounded Linear Operator, Spectrum of Bounded Linear Operators, Self Adjoint Bounded Linear Operators, Positive Operators, The Spectral Theorem

Text and Reference Books

1. L. Brown, *Elements of Functional Analysis***,** Von Nostrand and Renihold Company, 1970.
2. E. Kreyszig, *Introduction to Functional Analysis with Applications***,** John Wiley and Sons, New York, 1989.
3. A. Majeed, *Elements of Topological and Functional* Analysis, Ilmi Kitab Khana, 1994.
4. J. T. Oden, *Applied Functional Analysis*, Prentice Hall Inc, 1979.

Course Code: MATH-807 **Title:** Advanced Partial Differential Equations **Credit Hrs:** 03

Course Outline: Cauchy's Problems for Linear Second Order Equation in N- Independent Variables, Cauchy Kowalewski Theorem, Characteristic Surface, Adjoint Operations, Bicharacteristics, Spherical and Cylindrical Waves, Heat Equations, Wave, Laplace and Diffusion Equations in Spherical and Cylindrical Polar Coordinates, Maximum-Minimum Principle, Non-Homogeneous Partial Differential Equations.

Text and Reference Books

1. C. B. Chester, *Techniques in Partial Differential Equations*, McGraw Hill Book Company, 1971.
2. R. Dennemyer, *Introduction to Partial Differential Equations and Boundary Value Problems*, McGraw Hill Book Company, 1968.
3. H. F. Weinberger, *A First Course in Partial Differential Equations: with Complex Variables and Transform Methods*, Dover Publications, 1995.
4. E. C. Zachmanoglou and D. W. Thoe, *Introduction to Partial Differential Equations with applications*, Dover Publications, 1987.

Course Code: MATH-808 **Title:** Variational Inequalities **Credit Hrs:** 03

Course Outline: Variational Inequalities in Fixed Points, The Characterization of the Projection onto a Convex Set, A First Theorem about Variational Inequalities, Some Problems about Variational Inequalities, Variational Inequalities in Hilbert Space, The Obstacle Problem, Variational Inequalities for Monotone Operators, A Variational Inequality with Mixed Boundary Conditions, Penalization

Text and Reference Books

1. C. Baiocchi and A. Capelo, *Variational and Quasivariational Inequalities*, Applications to Free Boundary Problems, John Wiley and Sons, New York, 1984.
2. G. Isac, *Complementarity Problems and Variational Inequalities*, Springer-Verlag, 2006.
3. D. Kinderlehrer and G. Stampacchia, *An Introduction to Variational Inequalities and TheirApplications*,Academic Press, New York, 1980.
4. M. A. Noor, *Principles of Variational Inequalities*, Lambert Academic Publishing, Saarbrucken, Germany, 2009.

Course Code: MATH-809 **Title:** Convex Analysis **Credit Hrs:** 03

Course Outline: Affine Sets, Convex Sets and Cones, The Algebra of Convex Sets, Convex Functions, Functional Operations, Relative Interiors of Convex Sets, Closures of Convex Functions, Recession Cones and Unboundedness, Some Closedness Criteria, Continuity of Convex Functions, Separation Theorems, Conjugates of Convex Functions, Support Functions, Polars of Convex Sets,

Polars of Convex Functions, Linear Inequalities, Directional Derivatives and Subgradients, Differential Continuity and Monotonicity, Differentiability of Convex Functions, The Legendre Transformation, The Minimum of a Convex Function, Ordinary Convex Programs and Lagrange

Text and Reference Books

1. G. G. M. Ilyaev and V. M. Tikhomirov, *Convex Analysis: Theory and Applications*, AMS, 2003.
2. R. T. Rockafellar, *Convex Analysis*, Princeton University Press, 1997.
3. J. V. Tiel, *Convex Analysis: An Introductory Text, Wiley, 1984.*
4. J. B. H. Urruty and C. Lemarechal, *Fundamentals of Convex Analysis*, Springer Verlag, 2001.

Code: MATH-810 **Title:** Parameter Estimation and Sensitivity Analysis **Credit Hrs:** 03

Course Outline:

Some Basic Notions from Probability Theory, Probability spaces, random variables and distribution functions, Expected value and variance, Normally distributed random variables, Some simple examples, Parameter Estimation, Problem formulation, First order necessary condition, Second order conditions, The expected value for the least squares estimator,

The variance-covariance matrix for the least squares estimator Sensitivity Analysis, definition ,classical sensitivity, Sensitivity equations, fisher information matrix.

Text and Reference Books

1. J. N. Kapur, *Mathematical Modelling*, New Age International, 01-jan-1988.
2. Sandip Banerjee, *Mathematical Modelling*, CRC press, 07-Feb-2014.
3. J. Berry and J. Stephen, Mathematical Modelling, PLC Great Britian 1995.
4. E. A. Bender, *Mathematical Modelling*, 1st Edition, Dover Publications, 2000.

Course Code: MATH-811 **Title:** Semigroups in Geometric Functions Theory **Credit Hrs:** 03

Course Outline: Holomorphic Functions and their Convergence, Metric Spaces and Fixed Point Principles, Schwarz-Pick Lemma and Automorphisms, Boundary Behavior of Holomorphic Self-Mappings and Fixed Points and Fixed Point Free Holomorphic Self-Mappings, The Denjoy-Wolff Theorem, Commuting Family of Holomorphic Mappings, Hyperbolic Geometry and Fixed Points, The Poincare Metric and its Compatibility with Convexity, Infinitesimal Poincare Metric and Geodesics, Fixed Points of Non-expansive Mappings

One-Parameter Continuous Semigroup of Holomorphic and Nonexpansive Self-Mappings, Infinitesimal Generator, Nonlinear Resolvent and the Exponential Formula, Monotonicity w.r.t the Hyperbolic Metric, Flow Invariance Conditions, The Berkson-Porta Parametric Representation of Semi-Complete Vector Fields, Asymptotic Behavior of Continuous Flows, Stationary Points of a Flow, Null Points of Complete Vector Fields, Embedding of Discrete Time Group and Rates of Convergence of a Flow with an Interior Stationary Point, A Rate of Convergence in Poincare Metric, Continuous Version of the Julia-Wolff-Caratheodory Theorem, Lower Bounds for Monotone Functions, Asymptotic behavior of Continuous Flows, Dynamical Approach to Starlike and Spirallike Functions

Text and Reference Books

1. Shoket, *Semigroups in Geometric Function Theory*, Kluwer Academic Publishers, 2001.
2. S. G. Krantz, *Geometric Function Theory: Explorations in Complex Analysis*, 2006th Edition, Birkhäuser, 2005.
3. G. Kohr, G. Graham and I. Graham, *Geometric Function Theory in One and Higher Dimensions*, 1st Edition, CRC Press, 2003.
4. L. V. Ahlfors, *Conformal Invariants: Topics in Geometric Function Theory*, McGraw-Hill Inc., US, 1973.

Course Code: MATH-812 **Title:** Differential Subordination Theory and Applications

Credit Hrs: 03

Course Outline: History, Definitions and Some Results, Integral Operators, Introduction to the Theory of Second-Order Differential Subordinations, Applications of First-Order Differential Subordinations, First- Order Linear Differential Subordinations, Briot-Bouquet Differential Subordinations and its Applications in Univalent Functions, Generalized Briot-Bouquet Differential Subordinations, Analytic Integral Operators between Classes of Functions, Subordination-Preserving Integral Operators, Applications of Second-Order Differential Subordinations, Second-Order Linear Differential Subordinations, Operators Preserving Functions with Positive Real Part and Bounded Functions, Averaging Integral Operators, The Schwarzian and Starlikeness, Special Differential Subordinations, Higher Order Differential Subordinations, Introduction to Differential Subordinations of Several Complex Variables

Text and Reference Books

1. P. L. Duren, *Univalent Functions*, Grundlehren der Math. Wissenchaften, Springer-Verlag, New York-Berlin, 1983.
2. S.S. Miller and P.T. Mocanu, *Differential Subordination Theory and Applications*, Marcel Dekker, Inc., New York, Basel, 2000.
3. E. D. Rainville, *Special Functions*, Brett Macmillan Compnay, 1960.
4. 5. C. Viola, *Special Functions*, 1st Edition, Springer, 2016.

Course Code: MATH-813 **Title:** Conformal Mappings **Credit Hrs:** 03

Course Outline: Conformal Mappings on Simply-connected Domains: Mapping Properties of Analytic Functions, The Linear Transformations, The Schwarz Lemma, The Symmetry Principle, The Schwarz-Christoffel Formula, Domains Bounded by Circular Arcs, The Kernel Functions, Conformal Mappings of Nearly Circular Domains

Mapping Properties of Special Functions: Rational Functions of Second Degree, Exponential and Trigonometric Functions, Elliptic Functions, Domains Bounded by Arcs of Confocal Conics, The Schwarzian δ-Functions, the Elliptic Modular Functions

Conformal Mappings on Multiply-Connected Domains: Conanical Domains and their Characterizations by Extremal Problems, Green's Function and the Dirichlet Problems, Area Problems, The Kernel Functions and the Orthonormal Sets

Text and Reference Books

1. L. Bieberbach, *Conformal Mappings*, Chelsea Publishing Company, 1964.
2. Z. Nehri, *Conformal Mappings*, Dover Publications, 1975.
3. I. I. Gikhman and A.V. Skorokhod, *Theory of Random Processes*, N. Moskow, 1973.
4. R. Schinzinger and P. A. A. Laura, *Conformal Mappings*, Dover Publications, 2003.

Course Code: MATH-814 **Title:** Perturbation Methods **Credit Hrs:** 03

Course Outline: Approximate Solution of Linear Differential Equations, Approximate Solution of Nonlinear Differential Equations, Singular Perturbation Theory Perturbation Methods for Linear Eigen Value Problems, Asymptotic Matching Boundary Layer Theory, Mathematical Structure of Boundary Layers, Inner, Outer, And Intermediate Limits Higher-Order Boundary, Layer Theory Distinguished Limits and Boundary Layers Validity of the WKB Approximation Patched Asymptotic Approximations, WKB Solution of Inhomogeneous Linear Quations, Matched Asymptotic Approximation, Solution of the One-Turning-Point Problem.

Text and Reference Books

1. A. H. Nayfeh, *Perturbation Meth*ods, 1st Edition, Wiley-VCH, 2000.
2. E. J. Hinch, *Perturbation Methods*, Cambridge University Press, 1991.
3. A. H. Nayfeh and D. T. Mook, *Nonlinear Oscillations*, Wiley-VCH, 1995,
4. A. H. Nayfeh, *Introduction to Perturbation Techniques*, 1st Edition, Wiley-VCH, 1993.

Course Code: MATH-815 **Title:** Electro-dynamics **Credit Hrs:** 03

Course Outline: Maxwell's Equations, Electrodynamics Wave Equations, Boundary Conditions, Wave in Conducting and Non-Conducting Media, Reflection and Polarization, Energy Density and Energy Flux, Lorntz Formula, Wave Guides and Cavity Resonators, Spherical and Cylindrical Waves, Inhomogeneous Wave Equations, Retarded Potentials, Lenard-Wiechart Potentials, Fields of Uniformly Moving Point Charge, Radiation From a Group of Moving Charges, Field of Oscillating Dipole, Field of an Accelerated Point Charge.

Text and Reference Books

1. D. Corson and P. Lorrain, *Introduction to Electromagnetic Fields and Waves*, Freeman, 1962.
2. J. D. Jacson, *Classical Electromagnetic*, 3rd Edition, Wiley, 1998.
3. K. Panofsky and M. Philips, *Classical Electricity and Magnetism*, Addison Wesley, 1962.
4. I. R. Retiz and F. J. Malford, *Foundation of Electromagnetic Theory*, Addison Wesley, 1969.

Course Code: MATH-816 **Title:** Magneto-hydro-dynamics **Credit Hrs:** 03

Course Outline: Basic Equations: Equations of Electrodynamics, Equations of Fluid Dynamics, Ohm's Law Equations of Magneto Hydrodynamics.

Motion of an Incompressible Fluid: Motion of a Viscous Electrically Conducting Fluid with Linear Current Flow, Steady State Motion along a Magnetic Field, Wave Motion of an Ideal Fluid

Small Amplitude MHD Waves: Magneto-Sonic Waves, Alfven's Waves, Damping and Excitations of MHD Waves, Characteristic Lines and Surfaces.

Simple Wave and Shock Wave in Magneto Hydrodynamics, Stability and Structure of Shock Waves in Relativistic Magneto Hydrodynamics, Stability and Structure of Shock Waves, Discontinuities in Various Quantities, Piston Problem, Oblique Shock Waves.

Text and Reference Books

1. A. Etal, *Plasma Electrodynamics*, Pergamon Press, 1975.
2. H. Alfven, *Cosmical Electrodynamics*, Create Space Independent Publishing Platform, 1950.
3. J. E. Anderson, *Magneto hydrodynamics*, Shock Waves, M. I. T. Press, Cambridge, 1963.
4. T. G. Cowling, *Magnetohydrodynamics*, Intersience Publishers, 1963.

Course Code: MATH-817 **Title:** Fundamentals of Turbulence **Credit Hrs:** 03

Course Outline: Introduction to turbulence, various types of turbulent flows, transition to turbulence from laminar flow. Navier-Stokes equations and turbulence. Statistical Tools: Statistical moments and correlations, probabilities and averaging, space and time scales of turbulence. Kolmogorov's Theory of turbulence, Intermittency.

Text and Reference Books:

1. Marcel Lesieur, *Turbulence in Fluids*, 4th edition, Springer, 2008.
2. J. Mathieu and J. Scott, *An Introduction to Turbulent Flow*, 1st edition, Cambridge University Press, 2000.
3. A. Tsinober, *An Informal Conceptual Introduction to Turbulent Flows*, 2nd edition, Springer, 2009.
4. P. A. Durbin and B. A. P. Reif, *Statistical Theory and Modeling for Turbulent Flows*, 2nd edition, John Wiley and Sons, Ltd., 2011.

Course Code: MATH-818 **Title:** Lie Group Analysis of Differential Equations **Credit Hrs:** 03

Course Outline: Brief introduction to Lie group analysis of differential equations, preliminaries: heuristic approach in examples, finite differences and transformation groups in space of discrete variables, invariance of finite difference models, invariance difference models of ordinary differential equations, invariance difference models of partial differential equations, combined mathematical models and some generalizations, Lagrangian formalism for difference equations, Hamiltonian formalism for difference equations, symmetries and first integrals, discrete representation of ordinary differential equations with symmetries.

Text and Reference Books:

1. P.J Olver, *Application of Lie groups to Differential Equations*, Springer, New York, 1986.
2. S. Goldberge, *Introduction to Difference Equations*, John Wiley & Sons, New York, 1965.
3. V. Dorodnitsyn, *Applications of Lie Groups to Difference Equations*, CRC press, America, 2011.
4. A. Cohen, An introduction to the Lie theory of one-parameter groups, Ulan Press, 2012.
5. S. Elaydi, , *Introduction to Difference Equations*, 3rd edition, Springer, 2005.

Course Code: MATH-819 **Title:** Selected Topics in Applied Mathematics C**redits:** 3

Course Outline: Course out lines of this course will be determined by the concerned teacher as per requirement of a student in connection with his/her research.

Course Code: MATH-820 **Title:** Selected Topics in Pure Mathematics **Credit Hrs:** 03

Course Outline: Course out lines of this course will be determined by the concerned teacher as per requirement of a student in connection with his/her research.

Course Code: MATH-821 **Title:** Numerical Solutions of PDEs **Credit Hrs:** 03

Course Outline: Preliminaries, Classification of PDEs, Canonical Forms and Well-Posed Problems, Behavior of Solutions, Characteristics, Introduction to Finite Difference Methods
Basics of Finite Difference Approximations: Derivation of Finite Difference Approximations, Consistency, Stability, and Convergence for Difference Approximations, Lax Theorem, Matrix and Fourier Stability Analysis, Parabolic Equations: Explicit and Implicit Methods for the Heat Equation, Direction Splitting and ADI Schemes, Convection-Diffusion Equations
Hyperbolic Equations: Finite Difference Methods for the Wave Equation and High-Order Methods, First-Order Non-Linear Equations, Quasi-Linear and Conservation Forms, Characteristics, Shock Waves and Contact Discontinuities, Elliptic Equations: Dirichlet and Neumann Problems, Solvability, Direct vs. Iterative Methods of Solution

Text and Reference Books

1. R. J. LeVeque, *Finite Difference Methods for ODEs and PFEs*, SIAM, Philadelphia, 2007.
2. K. Morton and D. Mayers, *Numerical Solution of PDEs*, Cambridge University Press, 1994.
3. J. W. Thomas, *Numerical PDEs, Finite Difference Methods*, Springer-Verlag, 1995.
4. J. W. Thomas, *Numerical PDEs: Conservation Laws and Elliptic Equations*, Springer-Verlag, 1999.

Course Code: MATH-822 **Title:** Design Methods for Control Systems **Credit Hrs:** 03

Course Outline:
Introduction to Feedback Control Theory: Basic feedback theory, closed loop stability, stability robustness, frequency response design goals, loop shaping, limits of performance,
Classical Control System Design: Steady state error behavior, integral control, frequency response plots, classical control system design, lead, lag, and lag-lead compensation, the root locus approach to parameter selection, quantitative feedback theory,
Multivariable Control System Design: Poles and zeros of multivariable systems, MIMO structural requirements and design methods,
LQ, LQG, and H_2 Control System Design: LQ theory, LQG theory, H_2 optimization, feedback system design by H_2 optimization, examples and applications
Uncertainty Models and Robustness: parameter robustness analysis, the basic perturbation model, the small gain theorem, stability robustness of feedback systems, structured singular value robustness analysis, combined performance and stability robustness,

H_∞ – optimization and μ-synthesis: The mixed sensitivity problem, the standard H_∞ problem, suboptimal solutions and examples, state space solutions of standard H_∞ problem, optimal solutions to the H_∞ problem, integral control and high frequency roll-off, μ-synthesis with applications,

Text and Reference Books:

1. O. H. Bosgra, H. Kwakernaak, and G. Meinsma, *Lecture Notes on Design Methods for Systems and Control*, Dutch Institute of Systems and Control, 2006 .
2. B. Friedland, Control System Design: *An Introduction to State-Space Methods*, Dover Publications, 2005.
3. J. Distifano, A. R. Stubberud and L. J. Williams, *Schaum's Outline of Feedback and Control Systems*, 2nd Edition, McGraw-Hill Education, 2013.
4. R. F. Stengel, *Optimal Control*, Dover Publications, 1994.

Course Code: MATH-823 **Title**: Optimal State Estimation **Credit Hrs:** 03

Course Outline: Least squares estimation, Wiener filtering, propagation of states and covariances for discrete and continuous time systems, the discrete and continuous time Kalman filters, the H-infinity filters, the extended and unscented Kalman filters.

Text and Reference Books

1. D. Simon, *Optimal State Estimation*, John Wiley & Sons, Inc, 2006.
2. J. L. Crassidis and J. L. Junkins, *Optimal Estimation of Dynamic Systems*, 2nd Edition, Chapman and Hall, 2011.
3. B. M. Mohan and S. K. Kar, *Continuous Time Dynamical Systems:* State Estimation and Optimal Control with Orthogonal Functions, CRC Press, 2012.
4. R. F. Stengel*, Optimal State Estimation*, Dover Publications, 1994.

Code: MATH-824 **Title:** Linear Matrix Inequalities **Credit Hrs:** 03

Course Outline: Basic Concepts: Convex Optimization and LMIs, Dissipative Dynamical Systems: Storage functions and quadratic supply rates, Kalman-Yakubovich-Popov Lemma, The Positive Real Lemma, and Bounded Real Lemma, interconnected dissipative systems,
Stability and Nominal Performance: Review of Lyapunov stability,Generalized stability regions for LTI systems, the generalized plane concept, Quadratic, H-infinity, and H2 nominal performances,
Control Synthesis: Analysis to synthesis – a general procedure, Performance Specifications: H-infinity design, positive real design, H2 problem, State feedback problems, Discrete Time Systems,
Systems with Parametric Uncertainty: Affine parameter dependent systems, polytopic parameter dependent systems, Robust stability for autonomous systems: quadratic stability, quadratic stability of affine and polytopic models, parameter dependent Lyapunov functions,

Text and Reference Books

1. C. Scherer and S. Weiland, *Linear Matrix Inequalities in Control*, Dutch Institute of Systems and Control, 2006.
2. S. Boyd, L. E. Ghaoui, E. Feron and E. Balakrishnan, *Linear Matrix Inequalities in System & Control Theory, Society for Industrial & Applied*, 1994.
3. R. Bhatia, *Matrix Analysis*, 1997 Edition, Springer, 1996.
4. F. Zhang, *Matrix Theory*, 2nd Edition, Springer, 2011.

Course Code: MATH-825 **Title:** Stochastic Differential Equations **Credit Hrs:** 03

Course Outline: Brief Introduction to Diffusion Processes and How They Arise as Natural Approximations to Certain Discrete Processes, Review of Some Basic Facts about Brownian Motion and Martingales, ITO Stochastic Integrals Construction and Martingale Properties, ITO's Formula, Integration by Parts, Comparison with Stieltjes Integrals and Ordinary Calculus, Stochastic Differential Equations, Existence and Uniqueness of Solutions, Markov Property, Generators, Martingale Problem Characterization of Weak Solutions, Diffusion Processes, Dynkin's Formula, Calculations of Expectations and Probabilities, Converting to the Natural Scale, Stationary Distributions, Examples from Biology: Wright-Fisher Diffusions, Branching Diffusions

Text and Reference Books

1. K.L. Chung and R. J. Williams, *Introduction to stochastic integration*, Second Ed., Probability and its Applications, Birkhauser Boston Inc., MA, 1990.
2. S. Karlin and H. Taylor,*A Second Course in Stochastic Processes*, 1st Edition, Academic Press, 1981.
3. F. C. Klebaner, *Introduction to Stochastic Calculus with Applications*, 3rd Edition, Imperial College Press, 2012.
4. D. Richard, *Stochastic calculus: A practical introduction*, 1st Edition, CRC Press, 1992.

Course Code: MATH-826 **Title:** Fixed Point Theory and Applications **Credit Hrs:** 03

Course Outline: Lipschitzian, Contraction, Contractive and Non-Expansive Mappings, Banach's Contraction Principal with Application to Differential and Integral Equations, Brouwer's Fixed Point Theorem with Applications, Schauder's Fixed Point Theorem with Applications, Uniformly Convex and Strictly Convex Spaces, Properties of Non-Expansive Mappings, Extension's of Banach's Contraction Principal

Text and Reference Books

1. A. Granas and J. Dugundji, *Fixed Point Theory*, 2003 Edition, Springer, 2003.
2. R. P. Agarwal, D. O. Regan and D. R. Sahu, *Fixed Point Theory for Lipschitzian-type mappings with applications*, Springer-Verlag, USA.
3. R. P. Agarwal, *Fixed Point Theory and Applications*, Cambridge University Press, 2006.
4. V. I. Istratescu, *Fixed Point Theory: (Mathematics and Its Applications), 1st edition, 2001.*

Course Code: MATH-827 **Title:** Integral Inequalities **Credit Hrs:** 03

Course Outline: Some Quadrature Rules and Their Applications Ostrowski Inequality in L_1 And L_p- and L~ Spaces and Applications Gruss Inequality, Its Variant and Applications Ostrowski – Gruss Inequalities, Their Consequences and Applications Perturbed Results for Ostrowski and Ostrowski-Gruss Type Inequalities for Convex Functions, Hadmard, Sinequality, Hadmard,S Inequality for Convex Functions and Applications.

Text and Reference Books

1. S. Dragomir and T. M. Rassias, *Ostrowski Type Inequalities and Applications in Numerical Integration*, 2002 Edition, Springer, 2002.
2. G. A. Anastassiou, *Advanced Inequalities*, World Scientific Publishing Company, Singapore. 2011.
3. D. D. Bainov and P.S. Simeonov, *Integral Inequalities and Applications*, 1st Edition, Springer, 1992.
4. D. S. Mitrinovic, J. Picarcic and A.M Fink, *Inequalities Involving Functions and Their Integrals and Derivatives*, 1991 Edition, Springer, 1991.

Course Code: MATH-828 **Title**: Banach Algebras **Credit Hrs:** 03

Course Outline: Banach Algebra: Ideals, Homeomorphisms, Quotient Algebra, Wiener's Lemma, Gelfand's Theory of Commutative Banach Algebras: The Notions of Gelfand's Topology, Radicals Gelfand's Transforms.
Basic Properties of Spectra, Gelfand-Mazur Theorem and Symbolic Calculus of A-Valued Functions, Normed Rings, Gelfand-Naimark Theorem.

Text and Reference Books

1. M.A. Naimark, *Normed Algebras*, Wolters Noordhoff Publishing Groningen, Netherlands, 1972.
2. C. E. Rickart, *Banach Algebras*, D Van Nostrand Company Inc. NewYorK , 1960.
3. W. Rudin, *Functional Analysis*, McGraw Hill, Inc, New York, 1989.
4. W. Zelazko, *Banach Algebras*, American Elsevier Publishing Company Inc, New York, 1973.

Code: MATH-829 **Title:** Harmonic Functions Theory **Credit Hrs:** 03

Course Outline: Definition and Examples, lnvariance Properties, The Mean-Value Property, The Maximum Principle, The Poisson Kernel, The Dirichlet Problem, Converse of the Mean-Value Property, Real Analyticity and Homogeneous Expansions, Bounded Harmonic Functions Related Properties, Positive Harmonic Functions Related Properties, The Kelvin Transforms, Harmonic Polynomials, Harmonic Hardy Spaces, Harmonic Functions on Half-Spaces, Harmonic Bergman Spaces, The Decomposition Theorem, Annular Regions, The Dirichlet Problem and Boundary Behavior

Text and Reference Books

1. S. Axler, P. Bourdon, W. Ramey, *Harmonic Function Theory*,2n Edition, Springer, 2001.
2. S. Bergman,*The Kernel Function and Conformal Mapping*, American Mathematical Society, 1950
3. L. L. Helms, *Introduction to Potential Theory*, Wiley-Inter science, 1969.
4. O. D. Kellogg, *Foundations of Potential Theory*, Springer, 1929.

Course Code: MATH-830 **Title:** Cosmology **Credit Hrs:** 03

Course Outline: Review of Relativity, Historical Background, Astronomy, Astrophysics, Cosmology, The Cosmological Principle and its Strong Form, The Einstein and Desitter Universe Models, Measurement of Comic Distance, The Hubble Law and the Friedmann Models, Steady State Models, The Hot Big Bang Models, The Inicrowave Background, Discussion of Significance of a

Start of Time, Fundamentals of High Energy Physic, The Chronology and Composition of the Universe, Non-Brayonic Dark Matter, Problems of the Standard Model of Cosmology, Bianch Space-Times, Mixmaster Models, Inflationary Cosmology, Further Development of Inflationary Models, Kaluza-Klein Cosmologies, Review of Material.

Text and Reference Books

1. L. F. Abbott and S. Y. Pi, *Inflationary Cosmology*, World Scientific, 1986.
2. E.W. Kolb and M. S. Turner *The Early Universe*, Addison Wesley, 1990.
3. P. J. E. Peebles, *Principles of Physical, Princeton University Press*, 1993.
4. Ryan M. P. Jr. and Shepley, *Homogeneous Relativistic Cosmologies*, Princeton University Press, 1975.

Course Code: MATH-831 **Bifurcation and Chaos** **Credit Hrs: 3**

Course Outlines: Modeling with Nonlinear Systems of ODEs, Stability and Bifurcation Theory, The Hopf bifurcation and limit cycles, Lyapunnov exponents, Periodic and quasi periodic behavior Homoclinic & Heteroclinic Orbits and Mel'nikov Theory, Stability, Bifurcation and Chaos in I-dimensional Maps. Period doubling, Feigenbaum's Approach to Chaos, Properties of Chaos. The Lorenz Equations

Text and Reference Books:

1. S. Strogatz, *Nonlinear Dynamics, and Chaos: With Applications* to *Physics, Biology, Chemistry and Engineering*, 2nd Edition, Westview Press, 2014.
2. P. Glendinning, *Stability, Instability and Chaos*, 1st edition, Cambridge University Press, 1994.
3. R. Hillborn, *Nonlinear Dynamics and Chaos*, 2nd edition, Oxford University Press, 2001.
4. J. Ohtsubu, Stability, Instability and Chaos, 3rd edition, Springer, 2012.

Course Code: MATH-832 **Title:** Nonlinear Systems and Control **Credit Hrs:** 03

Course Outline:

Analysis techniques for nonlinear systems: phase portraits and their symmetries, singular points, phase plan analysis of linear and nonlinear systems, existence of limit cycles,

Fundamentals of Lyapunov Theory: Nonlinear systems and equilibrium points, concept of stability, linearization and local stability, Lyapunov's direct method and stability analysis, Krasovskii and variable gradient methods, performance analysis, control design based on Lyapunov's direct method

Advanced Stability Analysis: Stability of non-autonomous systems, Linearization and Lyapunov's direct methods for nonlinear autonomous systems, Asymptotic properties of functions and their derivatives, Barbalat Lemma, positive real and strictly positive real transfer functions, Kalman-Yakubovich lemma, passivity of linear systems,

Describing Function Analysis: Fundamentals of describing function analysis, common nonlinearities in control systems and their describing functions, Nyquist criterion and its extensions, existence and stability of limit cycles, reliability of describing function analysis,

Nonlinear Control Systems Design: Feedback Linearization: Canonical form, Lie derivatives and Lie brackets, diffeomorphism and state transformations, the Frobenius theorem, input-state and input-output linearization of SISO systems, the normal forms and zero dynamics, local and global

asymptotic stabilizations, tracking control and inverse dynamics, feedback linearization of multi-input systems,

Adaptive Control: Basic concepts in Adaptive Control, Adaptive control of first order systems, Adaptive control of linear systems of relative degree 1 and higher with full state-feedback and output feedback, Adaptive Control of nonlinear systems, robustness of adaptive control systems,

Text and Reference Books

1. H. K. Khalil, *Nonlinear Control*, Prentice Hall, 2002.
2. Slotine and Li, *Applied Nonlinear Control*, Prentice Hall, 1991.
3. A. Isidori, *Nonlinear Control Systems*, 3rd Edition, Springer, 1995.

B. Friedland, *Control System Design: An Introduction to State-Space Methods*, Dover Publications, 2005

External Evaluationof PhD Dissertation

i. The PhD dissertation must be evaluated by:
a. At least two external experts who may be either:

i. Pakistan-based Distinguished National Professors, Meritorious Professors; Professors from any national university or any Professor from top ranked universities by HEC; or Professor from any Pakistani University having minimum H-Index 30 for sciences, 15 for social science or 8 for Art & Humanities as determined by Web of Science. OR
ii. PhD experts from the world top 500 universities ranked by the Times Higher Education or QS World Ranking in the year corresponding to dissertation evaluation year.

OR
b. At least one external expert qualifying any one of the conditions mentioned at 'a' above if the PhD candidate publishes dissertation research in a peer reviewed journal that is classified by the HEC as category X or above.

4. Approval for the Updated Lists of External Examiners for Comprehensive Oral Exams of BS and MSc students, Thesis/Project Evaluation of BS, M.Sc, M.Phil and PhD Programs

4.1List of External Examiners for Comprehensive Oral Examinations, Projects/Thesis Evaluation of BS / M.Sc, MPhil/MS and PhD

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30. **Dr. Muhammad Taj**

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34. **Dr Saifullah Khalid**

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5. **Prof. Dr. Junqian Zhang**

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6. **Prof. Dr. Mehmet Avcar**

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7. **Prof. Dr. Huo Yongzhong**

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86. Prof. Chongqing Ru

Professor

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5. Scheme of Studies for BSc (Two Years Program) for affiliated colleges

There are three different courses of studies in Mathematics and each course has status of the subject

i) A-course of Mathematics (200 Marks)
ii) B-course of Mathematics (200 Marks)
iii) General Mathematics (200 Marks)

The following division is recommended:

5.1 **A-Course of Mathematics**

i) Differential and Integral Calculus (50 Marks)
ii) Complex Number and Analytic Geometry (50 Marks)
iii) Infinite Series, Differential Equation and Laplace Transform (50 Marks)
iv) Linear Programming and Application of Calculus(50 Marks)

5.2 **B-Course of Mathematics**

i) Group Theory and Linear Algebra (50 Marks)
ii) Number Theory , Topology and inner product space(50 Marks)
iii) Vector Analysis and Statics (50 Marks)
iv) Numerical Method and Dynamics(50 Marks)

5.3 **General Mathematics**

i) Complex Number and linear Algebra and Analytic Geometry (50 Marks)
ii) Differential and integral calculus (50 Marks)
iii) Application of calculus and Analytical Geometry of Three Dimension (50 Marks)
iv) Numerical Method , Infinite Series , Linear Programming and Differential Equation(50 Marks)

5.4 **Course Contents of A-Course of Mathematics**

5.4.1 **Paper-I : DIFFRENTIAL AND INTEGRAL CALCULUS (3rd Year)**

Students have to attempt five **question** out of eight: three from Section-I and two question from Section-II

Section-I Differential Calculus (5 out of 8)

A review of real number system, upper and lower bounds, Function and their Graphs , limits , continuity and related theorems , Tangents and the Derivative at a point, Differential rules and their Application , Derivative as a rate of change , Derivatives of trigonometry ,exponential, logarithmic hyperbolic function and differentiation of their inverse , Implicit differentiation, Related rates, Linearization and differentials , Higher derivatives , Leibnitz's theorem ,Rolle's theorem ,Lagrange mean value theorem , increasing and decreasing function , Cauchy's mean-value theorem , indeterminate forms and L'Hospital 's rule.

Section-II Integral calculus (3 out of 8)

Area and estimating with finite sums, sigma notations and limit of finite sums, definite integral as the limit of a sum, properties of definite integral, fundamental theorem of calculus, indefinite integrals

and techniques of integration, reduction formulae, application of definite integral to area, arc length and other problems

Recommended Books

i) G.B.Thomas Jr.M.D.Weir and J.R.Hass, Thomas Calculus , 12th edition ,Pearson Edu.Inc.,2010.
ii) H.Anton, Calculus.(Latest edition) . John Wiley and sons , New York .
iii) S.M.Yousaf ,Calculus
iv) Zia-ul-Haq , Calculus and analytical Geometry , Carvan Book , 2001.
v) C.H.Edwards and D.E.Penny, Calculus and analytical Geometry, (Latest Edition). Prentice Hall, Inc.

5.4.2 **Paper-II Complex Number and Analytical Geometry (3rd year)**

Student have to attempt five question out of eight: two from section-I and three question from section-II

Section-I: Complex Numbers (3 out of 8)

Complex Numbers and algebra of complex number, polar representation, Euler's Formula,De-Moivres' theorem and its applications, Trigonometric and Hyperbolic function, Exponential and logarithmic function, Separation of complex valued functions and imaginary parts, Summation of series.

Section-II: Analytical Geometry (5 out of 8)

Two-dimensional Analytical Geometry (2 out of 8)

Translation and rotation of axes, General equation of the second degree and the classification of conic sections, Conic sections in polar coordinates, Tangents and normals, Pedal equation of curves, Tracing of polar curves.

Three- dimensional Analytical Geometry (3 out of 8)

Direction cosines and ratios, angle between two lines, Standard forms of equations of lines and planes, Distances between points, Lines and planes, Spherical polar and cylindrical coordinate systems, Standard form of the equation of sphere, Cylinder, Cone,ellipsoid, paraboloid and hyperboloid, symmetry, intercepts and sections of a surface, tangent planes and normals.

Recommended Books:

i) G.B.Thomas Jr.M.D.Weir and J.R.Hass, Thomas Calculus, 12th Edition, Pearson Edu. Inc. 2010.
ii) H.Anton, Calculus.(Latest edition) . John Wiley and Sons, New York.
iii) S. M. Yousaf, Muhammad Amin, Calculus with Analytical Geometry.
iv) Zia-ul-Haq , Calculus and analytical Geometry , Carvan Book , 2001.
v) C.H.Edwards and D.E. Penny ,calculus and analytical Geometry,(Latest edition). Prentice Hall,Inc.
vi) E.H. Swokowski, Calculus with Analytical Geometry ,(Latest Editor). PWS publishers, Boston, Massachusetts.

5.4.3 **PAPER.III: INFINITE SERIES, DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS (4th Year)**

Students have to attempt five questions out of eight:two from section-I and three question from section-II

Section –I: Infinite serious (3 out of 8)

Sequence of numbers and their convergence, Algebra of convergent sequences, Infinite series and their convergences, Convergences tests for infinite series: Comparison, quotient, ratio, root and integral tests, Absolute and conditional convergences, Interval and radius of convergences, Taylors and Maclaurin's theorems in finite and infinite form and their use' in expansion of function.

Section-II: Differential Equations (5 out of 8)

Definition and types of differential equations and their formations, Different methods f solving first order ordinary differential equations, The Bernoulli, Ricatti and Clairaut equations, Families of curves, Orthogonal trajectories, Initial and boundary value problem, Application of first order differential equations in problems of decay and growth of populations of dynamics and logistics, second and higher order linear differential equations with constant coefficients and methods of their solutions, Cauchy-Euler equations, system of second order linear differential Equations,Method of undetermined coefficient ,Method of variation of parameters,Reduction of order,Laplace transforms and applications.

Recommended Books

1**.** G.B.Thomas Jr.M.D weir and J.R Hass, Thomas Calculus,12th Edition, pearson Edu. Lnc.,2010.
2. D.G Zill and M.R. Cullen, Differential Equations with boundary –Value problems, 3rd Edition, PWS publishing Company, 1997.
3. H.Anton, Calculus.(Latest Edition).Jhon Wiley and Sons, New York.
4. S.M. Yosaf, Mathematical Methods
6. Zia ul Haq, Calculus and Analytical Geometry, The Carvan Book House,2001.
8. E.H Swokowski, Calculus with Analytical Geometry (Latest Edition).PWS publishers,boston, Massachusetts

5.4.4 **PAPER-IV: LINEAR PROGRAMMING AND APPLICATIONS OF CALCULUS (4th Y ear)**

Students have to attempt five questions out of eight :three from section-l and two from section-ll.

Section-l: (5 out of 8)

(a) Applications of Differential Calculus 4/5

Curves and their Cartesian ,polar and parametric representations, Asymptotes ,Maxima and Minima, points of inflexion and their applications, singular points, curve tracing, Curvature ,centre and radius of curvature, Functions of several variables, limits continuity and partial derivatives, Maxima and minima of functions of two variables with applications ,Approximations ,Equations of tangent plane and normal line to a surface.

(b) Linear programming 1/5

Introduction to Operations Research in general and in particular to linear programming simplex method ,Assignment Models

Section-ll: Applications of Integral Calculus (3/8)

Rectifications and Quadrature, Simple cases of double and triple integrals,Volumes and area of surfaces of revolutions.

Books Recommended

G.B Thomas Jr. M.D weir and J.R. Hass, Thomas Calculus, 12th Edition,pearson Edu. Inc., 2010.
2. A.Sultan, Linear programming, Academic press.
3. W.A Spivey Linear programming ,McMillan Co.
4. Hamday A. Taha, Operations Research
5. Hiller, Introduction to Operations Research
6.Dar,K.H. Mathematical Techniques, Carvan Book House,2001.

5.5 **B-COURSE OF MATHEMATICS**

5.5.1 **PAPER-I: GROUP THEORY AND LINEAR ALGEBRA (3rd Year)**

Students have to attempt five questions out of eight :three from section-l and two from section-ll.

Section-l Group Theory (3 out of 8)

Definitions and examples of groups, Groups of residue classes, Cyclic group, Order of a group and order of an element of a group ,subgroup, cosets, Lagrange's theorem and its applications Permutations ,even and odd permutations ,Cycles and length, transpositions

Section-ll: Linear Algebra (5 out of 8)

Fields, Vector spaces,subspaces and examples ,Linear dependence and independence,Bases and dimensions ,Linear transformation,Motivations of ,matrices through a system of linear homogenous and non -homogenous equations , Elementary row and column operations on matrices ,Algebra of matrices Determinants of matrices ,their properties and evaluation of various kinds of matrices ,Matrix of a linear transformation , Rank of a matrix , Evaluation of ranks and inverses of matrices ,solution of system of homogenous and non- homogenous linear equations (Elimination and Gauss Method)

Books Recommended

i) H.Anton, Elementary Linear Algebra .(Latest Edition).J. Wiley
ii) S.M. Yosaf, Mathematical Methods
iii) A.Majeed .Group Theory.
iv) K.L. Mir, Linear Algebra, ilmi kutab khana.
v) C.H Edwards, Jr. and D.E. Penney, Elementary Linear Algebra,(Latest Edition).prentice Hall, International edition
vi) S.J.Axler, Linear Algebra, Done Right, Springer-Verlag,1996

5.5.2 **Paper-II: NUMBER THEORY, TOPOLOGY AND INNER PRODUCT SPACE(3rd Year)**

Student have to attempt five question out of eight: two from section-I and three question from section-II

Section-I: Number Theory (3 out of 8)

Divisibility Euclid's theorem (Division Algorithm) ,Greatest common divisor and latest common multiple, theory of primes, Linear Equations, Diophantine Equation.

Section-II: Topology & Inner Product Space (5 out of 8)

Definition and examples of metric space, open balls, open sets in a metric space, interior, exterior, boundary and closure of a set in a metric space, definition and example of topology and topological spaces, open and closed sets in topological spaces, Neighborhood, Limit Point, Interior, exterior, boundary and closure of sets in a topological spaces, Definition and example of inner product spaces, Orthogonality, orthogonal and orthogonal system, orthogonal matrices.

Books Recommended

i) A. Majeed, Element of topology and functional Analysis, Ilmi kitab Khana, Lahore 1997.
ii) S. M. Fahfa, Introduction to point set topology.
iii) B. Ahmad, General Topology, 1998.
iv) S.Manzur Hussain, Introduction to theory of Number . G. A Jones and J.M.Jones,
v) Elementary Number Theory, Springer-Varlog, London Limited, 1998.
vi) M. B. Nathanson, Methods in Number Theory, Springer-Verlog, New York,2000.
vii) Introductry set topology by S. M. Yhaya.

5.5.3 **PAPER-III: VECTOR ANALYSIS AND STATICS (4th YEAR)**

Students have to attempt five questions out of eight: two from section-I and three from section-II

Section-I: Vector Analysis (3 out of 8)

Vectors and there algebra , coordinate systems and bases, Scalar and vector triple products, Differentiation and integration of vectors, Scalar and vector point functions, Concepts of gradient, divergence and curl along with their applications.

Section-II: Statics (5 out of 8)

Composition and resolution of forces, Particles in equilibrium, Parallel forces, moments, Couples, Geneal conditions of equilibrium of coplanar forces, Principle of virtual work, Friction, Centre of gravity.

Books Recommended

1. G.B. Thomas Jr. M.D. Weir and J.R. Hass, Thomas Calculus, 12th Edition, Pearson Edu. Inc.,2010
2. Q. K. Ghori, Introduction to Mechanics (West Pakistan Co., Ltd., Lahore)
3. G.D. Smith, Vector Analysis, (Latest Edition), Oxford University Press.
4. K.L.Mir, Vector Analysis, (Latest Edition), Ilmi Kitab Khana.
5. M.N. Talpur, Calculus with Analytical Geometry.
6. L.Synge and B.A Griffith, (Latest Edition), Principles of Mechanics. Mc-Graw Hill.
7. R. Whitworth and Dyke, Guide to Mechanics, (Latest Edition), MacMillan

5.5.4 **PAPER-IV NUMERICAL METHODS AND DYNAMICS (4th Year)**

Students have to attempt five questions out of eight : three from section-I and two questions from section-II.

Section-I: Numerical Methods (3 out of 8)

Introduction to Numerical Analysis, Numerical solution of algebraic and transcendental equations: graphical method, bisection method, iteration method, Newton-Raphson method, secant method and method of false position, S ystem of Linear equations: Guass-siedel and Jacobi methods, Numerical integration: Trapeziodal and Simpsom's rules. (If possible computer programming may be used for problem solving).

Section-II: Dynamics of a Particle (5 out of 8)

Motion in a straight line, Uniformly accelerated and resisted motion. Velocity, acceleration and their components in cartesian and polar coordinates, tangential and normal components, Conservative forces, Projectiles, Central forces and orbits, Simple harmonic motion, Damped and forced vibrations.

Books Recommended

1. Robert-W.Horn beck, Numerical Methods, Quantum Publishers.
2. Alestair Wood, Introduction to Numerical Analysis, Addison Wesley.
3. M. Iqbal, Numerical Analysis, National Book Foundation.
4. S.A. Bhatti, N.A. Bhatti, Numerical Methods
5. Q. L. Ghori, Introduction to Mechanics (West Pakistan Publishing Co., Ltd., Lahore).

5.6 **GENERAL MATHEMATICS**

5.6.1 **PAPER-I: COMPLEX NUMBER, LINEAR ALGEBRA AND ANALYTICAL GEOMETRY (3rd Year)**

Students have to attempt five questions out of eight: two from section-I and three questions from section-II.

Section-I: Complex Number System (3 out of 8)

Real number system and properties of real numbers, Real-valued functions and their graphs, Complex numbers their algebra and Polar form, De-Moivres' theorem, n n^{th} roots of complex numbers, Complex functions, Sum of trigonometric series.

Section-II: Linear Algebra and Analytical Geometry (5 out of 8)

Matrices: rank and inverse of a matrix. Linear transformation and their matrices, Determinants,
System of linear equations, Analytic Geometry of two Dimension, Translation and rotation of axis, Properties of tangents and normal, Polar equation of conics Pedal equation, Tracing of polar curves.

5.6.2 **PAPER-II: DIFFERENTIAL AND INTEGRAL CALCULAS (3rd Year)**

Students have to attempt five questions out of eight: three from section-I and two questions from section-II

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Section-I: Differential Calculus (5 out of 8)

Techniques of finding limits, Continuity of a function, Differentiability, Indeterminate forms, Use of Rolle's theorem, Mean value theorems (Lagrange and Cauchy), Taylor and Maclaurins series, Derivatives, Higher Derivatives and Partial Derivatives, Related rates.

Section-II: Integral Calculus (3 out of 8)

Techniques of integration, Definite integral as limit of a sum, Evaluation of a definite integral by definition, Improper integrals, reduction formulae.

5.6.3 PAPER-III: APPLICATION OF CALCULUS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS (4th Year)

Students have to attempt five questions out of eight: two from section-I and three questions from section-II.

Section-I: Application of differential and Integral Calculus (5 out of 8)

Asymptotes, maxima and minima of a function of one and two variables, Curvatures and centre of curvature, rectification, quadrate, Eulers theorem, Chain Rule, Total derivative, Equation of tangent, Plane and normal lines to surfaces volumes and surface area of revolution, Simple cases of double and triple integrals.

Section-II:: Analysis Geometry (3 out of 8)

Direction cosines and ratios, Angle between two lines, Standard form of equations of planes and lines, Intersection of planes and lines, Distance between points, lines and planes, Spherical, polar and cylindrical coordinate systems, Standard form of the equations of a sphere, cylinder, cone, ellipsoid, paraboloid and hyperboloid, Symmetry and intercepts of a surface, Tangent planes and normal.

5.6.4 PAPER-IV: NUMERICAL METHODS, INFINITE SERIES, LINEAR PROGRAMMING AND DIFFERENTIAL EQUATIONS (4th Year)

Students have to attempt five questions out of eight: two from section-I and three questions from section-II.

Section-I: Numerical Methods and Infinite Series (3 out of 8)

Introduction to infinite series and tests for their convergence, Absolute and conditional convergence. Introduction to Numerical Analysis, Numerical Solution of algebraic and transcendental equations: bisection method, Newton-Raphson method.

Section-II: Linear Programming and Differential Equations (5 out of 8)

Introduction to linear programming, Simplex methods and their examples from real life, Differential equations of first order, Separable, Homogenous equation, Exact equation, Linear differential equation, Bernoulli's equation, orthogonal trajectories, Differential equations 2nd and higher order Cauchy Euler equation, Method of variation of parameters, Method of undetermined Cofficient.

Books Recommended for General Mathematics (Papers I to IV)

1. G.B. Thomas Jr. M.D. Weir and J.R. Hass, Thomas Calculus, 12th Edition, Pearson. Edu. Inc., 2010.
2. S.T. Tan, Applied Mathematics. For the Managerial, life, and social sciences.
3. H. Anton, Elementry Linear Algebra. (7th edition, 1997). Wiley.
4. H. Anton, Calculus, (Latest Edition)m John Wiley and Sons, New York.
5. E. Kreyosing, Advanced Engineering Mathematics, (Latest Edition), J. Wiley.
6. M. Iqbal Numerical Analysis. (Latest Edition), National Book Foundation.

7. Fiaz Ahmad and M.A. Rana, Elements of Numerical Analysis, (Latest Edition), NBF.
8. S.M. Yousaf, Mathematical Methods.
9. Hmaday A. Taha, Operations Research.
10. A. Sultan, Linear Programmingm, Academic Press.

Other Books

1. Calculus S.M. Yousaf
2. Introduction to Mechanics, S.M. Yousaf
3. Topology Ch. M. Amin
4. Introductions Set Topology S.M. Yousaf.
5. Metric Spaces by Z.R Bhatti
6. Elementary Theory of Numbers by Sayyed Manzoor Hussain
7. Elementary Numerical Analysis by Dr. M. Iqbal
8. Vector Analysis by Dr. Munawwar Hussain.

Any Other Item